Rotational Model for Oscillating Waves Over the Surface of A Sphere

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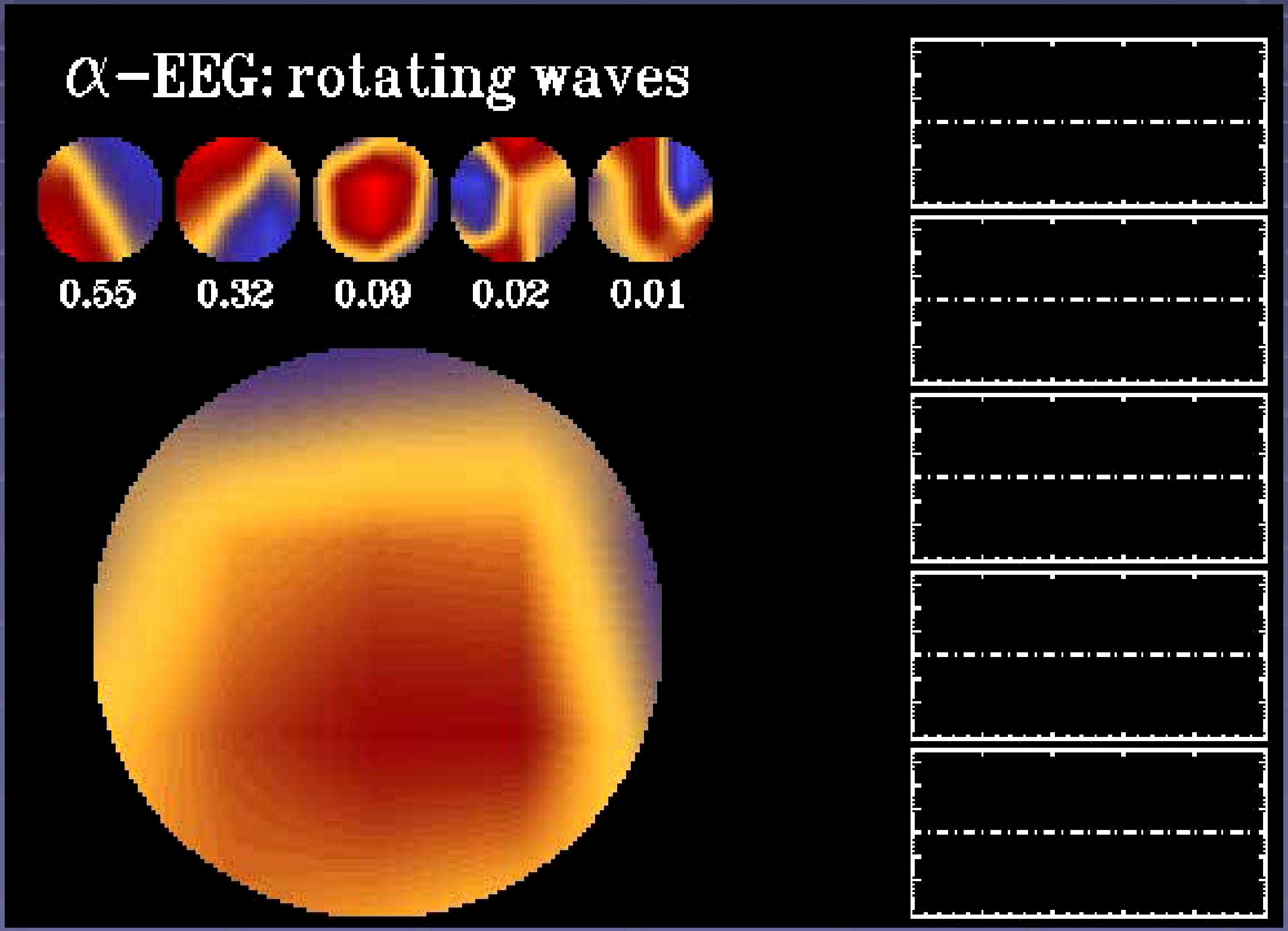
Introduction

- Wave oscillations in coupled networks find themselves in different biological environments
- Ex: Brain Wave Activity, Calcium deposits on frog oocyte surfaces
- Oscillations can be captured on electroencephalograms (EEGs)

Wave Rotation in EEGs



Mathematical Background

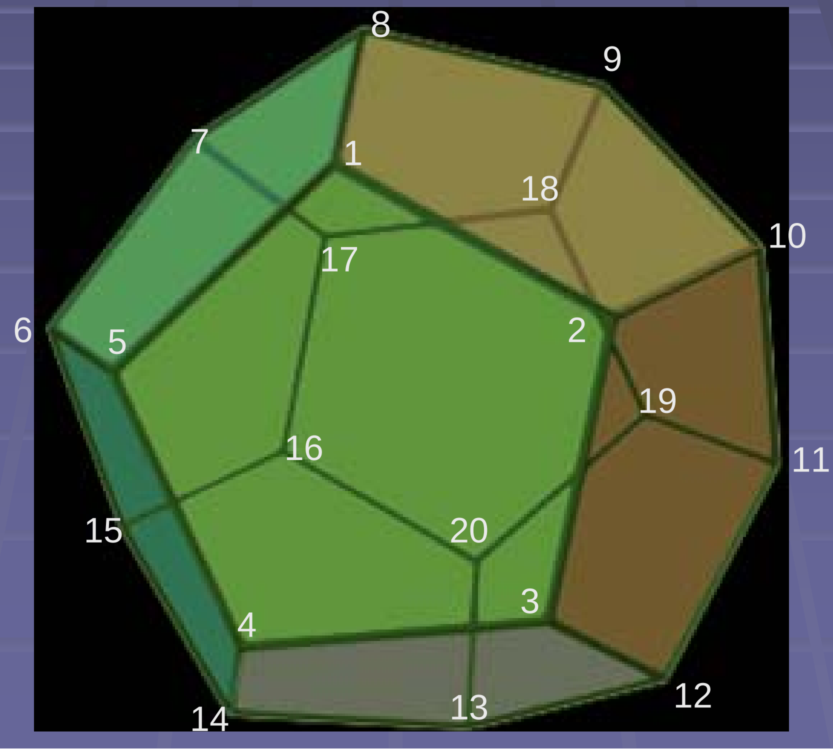
- Weakly-coupled limit cycle oscillators reduced to scalar equations
- Must Prove to analyze data

Research Goals

- Oscillators tend toward synchronous movement
- To create a model that has non-synchronous solutions
- To determine how often these non-synchronous solutions occur in differing models (n=20 and n=64 oscillators)

Dodecahedron: A Simple Model

- Is a good model to start with:
  - 20 points evenly distributed on surface
  - Exactly 3 neighbors for each of the vertices



Dodecahedron: A Simple Model

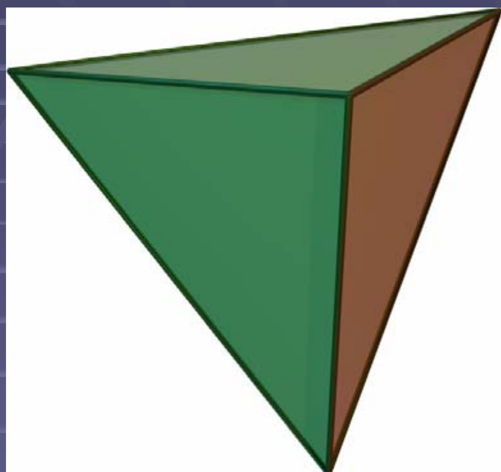
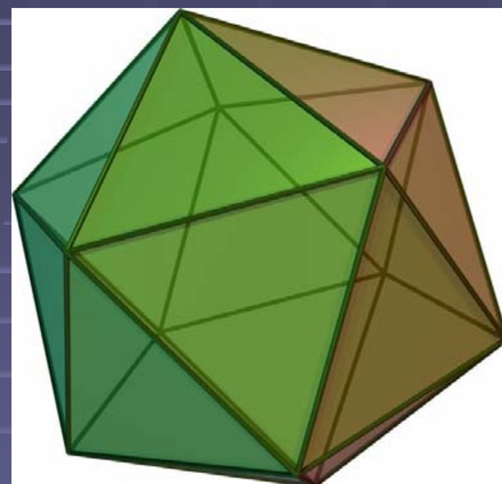
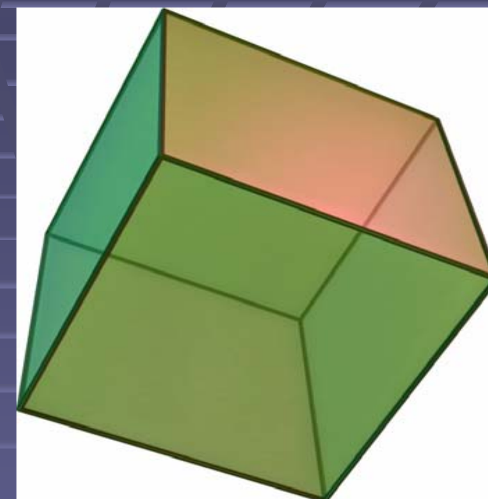
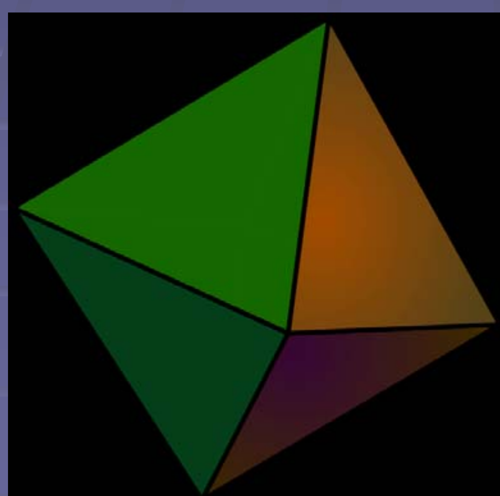
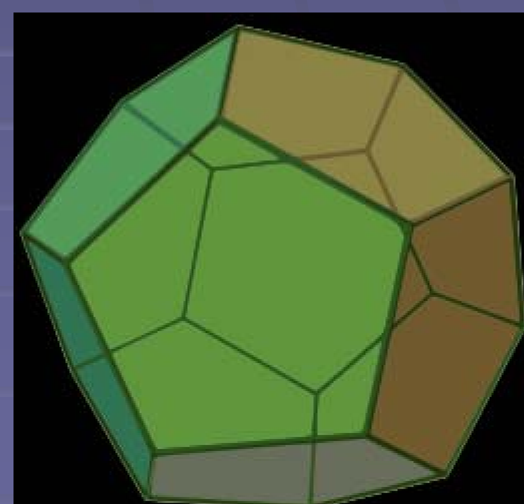
- x1p=c+H(x8-x1)+H(x5-x1)+H(x2-x1)
- x1'=-x1
- x2'=c+H(x10-x2)+H(x3-x2)+H(x1-x2)-x1p
- x3'=c+H(x12-x3)+H(x4-x3)+H(x2-x3)-x1p
- x4'=c+H(x14-x4)+H(x5-x4)+H(x3-x4)-x1p
- x5'=c+H(x6-x5)+H(x4-x5)+H(x1-x5)-x1p
- x6'=c+H(x15-x6)+H(x7-x6)+H(x5-x6)-x1p
- x7'=c+H(x17-x7)+H(x8-x7)+H(x6-x7)-x1p
- x8'=c+H(x9-x8)+H(x7-x8)+H(x1-x8)-x1p
- x9'=c+H(x18-x9)+H(x10-x9)+H(x8-x9)-x1p
- x10'=c+H(x11-x10)+H(x9-x10)+H(x2-x10)-x1p
- x11'=c+H(x19-x11)+H(x12-x11)+H(x10-x11)-x1p
- x12'=c+H(x13-x12)+H(x11-x12)+H(x3-x12)-x1p
- x13'=c+H(x20-x13)+H(x14-x13)+H(x12-x13)-x1p
- x14'=c+H(x15-x14)+H(x13-x14)+H(x4-x14)-x1p
- x15'=c+H(x16-x15)+H(x14-x15)+H(x6-x15)-x1p
- x16'=c+H(x20-x16)+H(x17-x16)+H(x15-x16)-x1p
- x17'=c+H(x18-x17)+H(x16-x17)+H(x7-x17)-x1p
- x18'=c+H(x19-x18)+H(x17-x18)+H(x9-x18)-x1p
- x19'=c+H(x20-x19)+H(x18-x19)+H(x11-x19)-x1p
- x20'=c+H(x19-x20)+H(x16-x20)+H(x13-x20)-x1p

H(x)=sin(x)

Dodecahedron: A Simple Model

- Theorem (from an earlier Ermentrout paper):
  - Let H(x) be an odd periodic function:
    - H(x+2*pi) =H(x)
    - H(-x) = -H(x)
  - If these functions are weakly coupled in a network, then there exists a real world solution to this function that is asymptotically stable
- So if we use this model, we know there exist non-synchronized asymptotically stable solutions

Platonic Solids

N=64 Model

- Difficult to evenly distribute 64 points about a sphere
- Neil Sloane-algorithm for finding coordinates of evenly distributed points on sphere’s surface
- C program created to find a neighborhood of fixed radius around each point
- Generated connection matrix (5-7 neighbors)

Results

- Our goal was to find out if non-synchronous solutions exist, and if so, how often they occur
- For the Dodecahedron Model, ~2% of the time non-synchronous solutions
- For the n=64 oscillator model, ~10% of the time non-synchronous solutions

Results (cont.)

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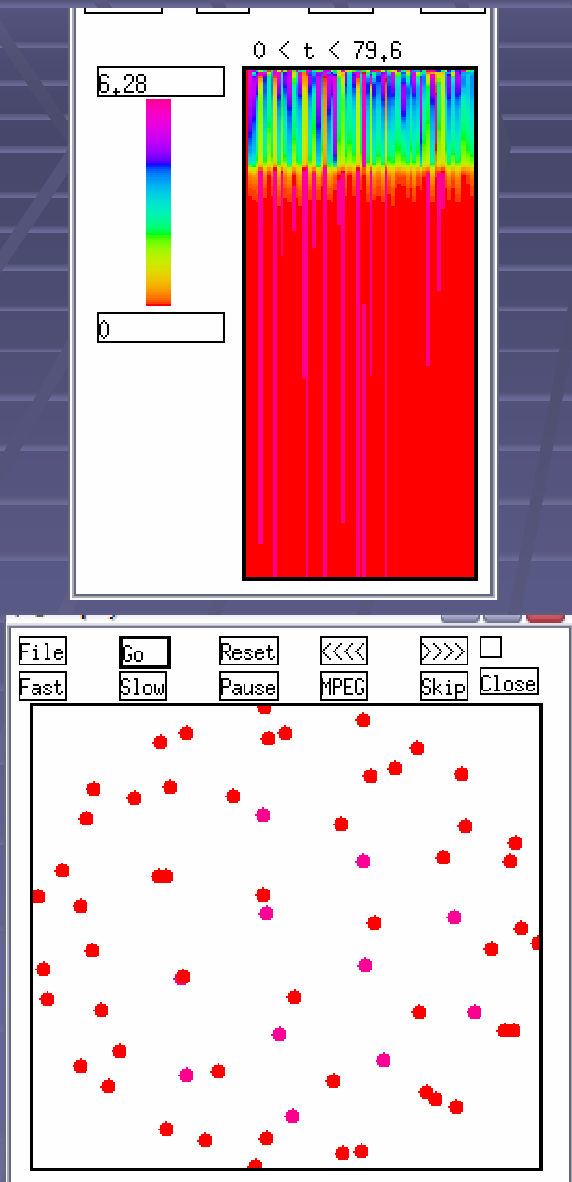
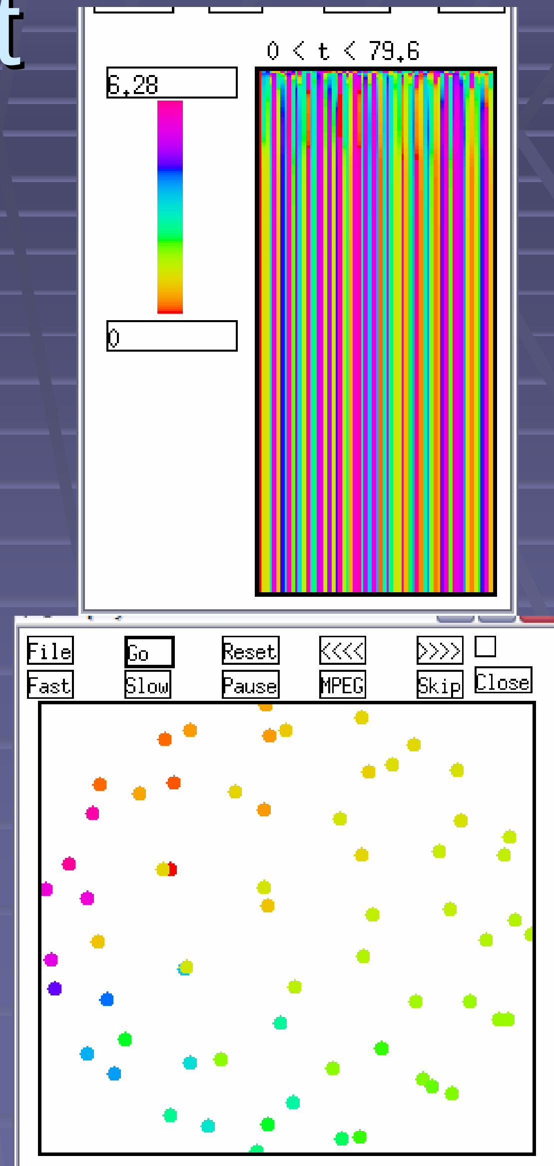
Time	X0	X1	X2	X3	X4	X5	X6	X7
100	0	6.283185	6.283185	6.283185	5.403867e-16	6.283185	6.283185	6.283185
100	0	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185
100	0	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185
100	0	0.7742215	1.148241	1.947981	5.107681	2.843709	0.8313545	1.519726
100	0	9.024043e-16	1.010218e-15	1.703041e-15	8.157759e-16	1.332929e-15	1.11914e-15	1.563914e-15
100	0	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185
100	0	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185
100	0	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185	6.283185
100	0	1.085755	1.613739	2.980651	4.972976	3.606308	0.6875429	2.489145
100	0	6.035822e-15	5.764986e-15	9.391871e-15	6.283185	4.601921e-15	5.371261e-15	8.890606e-15
100	0	6.283185	6.283185	6.283185	2.248231e-15	6.283185	6.283185	6.283185
100	0	7.50973e-15	4.895275e-15	8.420039e-15	6.283185	6.283185	6.101004e-15	9.56792e-15
100	0	4.095285e-15	4.466331e-15	6.935234e-15	6.283185	4.906239e-15	3.958816e-15	6.336197e-15
100	0	9.005454e-16	1.011865e-15	1.704152e-15	8.108106e-16	1.338943e-15	1.117993e-15	1.563258e-15
100	0	0.9579977	0.8454016	1.729876	5.891264	0.6696078	4.139472	2.782176
100	0	5.195515	5.378406	3.566091	0.5776515	5.782584	1.836276	2.700585
100	0	9.273564e-17	1.674152e-16	3.731395e-16	1.233485e-16	2.991039e-16	1.168475e-16	3.094867e-16
100	0	0.7742215	1.148241	1.947981	5.107681	2.843709	0.8313545	1.519726
100	0	6.283185	6.283185	6.283185	1.346695e-15	6.283185	6.283185	6.283185
100	0	6.283185	6.283185	6.283185	1.049936e-13	6.283185	6.283185	6.283185

Synchronous vs. Non-synchronous Movement





Conclusions

- We found a successful model that produced non-synchronous solutions
- We have an idea of how often these solutions occur for two different models

Future Work

Future research on this topic can focus on:

- Observing higher numbers of n oscillators on the sphere’s surface
- Utilizing different periodic functions other than sin(x)
- Placing not only oscillators but excitable or other active elements
- Altering the connectivity of networks to see if this affects the probability of synchronization
- Addition of long range connections with delays to see its effect (Connections common in the brain)

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Thank you.

Any Questions?