Rotational Model for Oscillating Waves Over the Surface of A Sphere

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Abstract

The neurons of the brain communicate by electrical signals known as action potentials. For signals to propagate, separate parts of large neural networks must fire in different patterns. This results in waves that oscillate over the surface of the brain. Oscillators tend to synchronous movement, where each oscillator in the network ultimately mimics every other oscillator. Our model of coupled wave oscillators determines what percentage of time non-synchronous movement is observed over the surface of a sphere.

Introduction

Wave oscillations in coupled networks find themselves in different biological environments. One example is brain wave activity, and another is calcium deposition on the surface of frog oocytes. This project is geared towards finding a model that determines how often and what type of synchronous movement is observed on a sphere's surface.

Electroencephalograms (EEGs) can capture the rotations of electrical oscillators (Figure 1). Modeling the movement of these oscillating waves would be useful in better understanding information processing in the brain. A good model for a network of this type is one in which there are n oscillating points on a sphere, that are coupled to each of its neighbors.

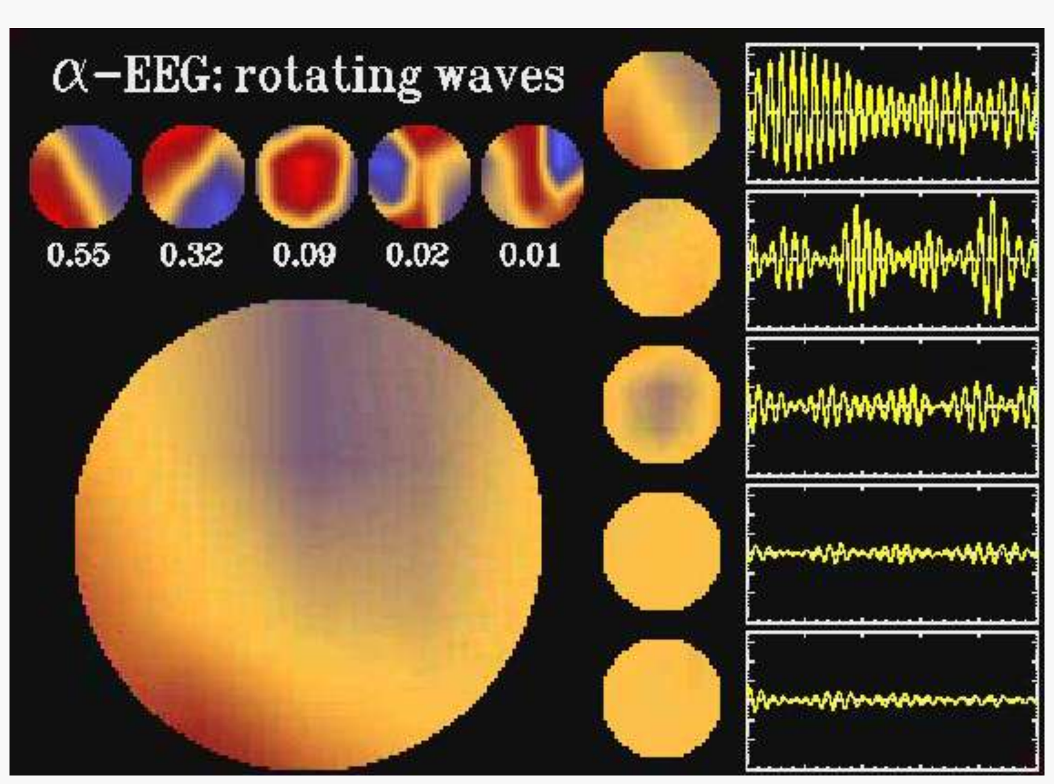


Figure 1. Rotating Waves in an EEG

Method

1. Periodic Phase Equations based on sin(x) were written for each oscillator at the dodecahedron's vertices
2. Weakly coupled limit cycles were proven to be reduced to coupled scalar equations
3. XPPAut used to find the percentage of synchronous movement of coupled oscillators at the vertices of the dodecahedron
4. A spherical coding website was used to find coordinates for evenly displaced points on a sphere's surface
5. A C program was created to designate neighbors within a fixed radius
6. A connection matrix was generated from the program's results
7. XPPAut used again to determine non-synchronous movement for a sphere with N=64 points

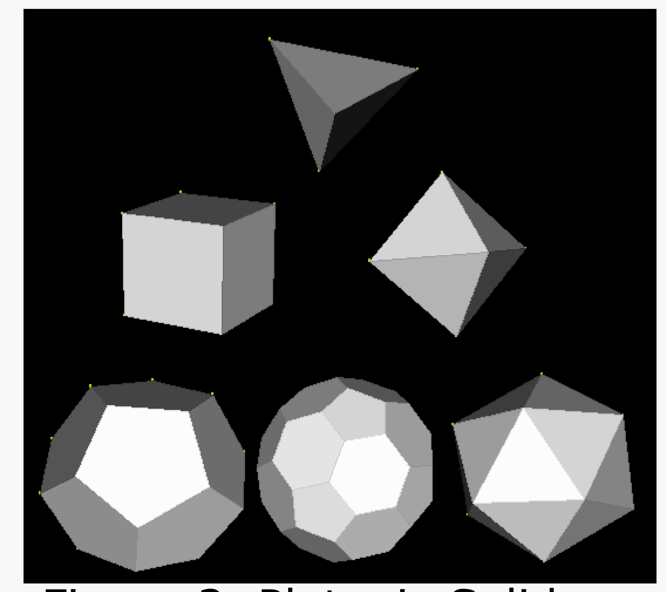
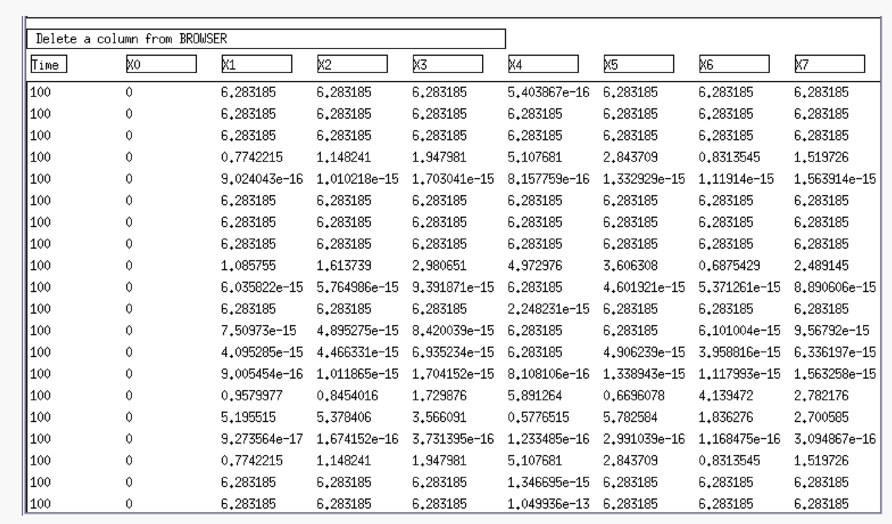


Figure 2. Platonic Solids



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Time	X0	X1	X2	X3	X4	X5	X6	X7
100	0	6,283185	6,283185	6,283185	5,403867e-16	6,283185	6,283185	6,283185
100	0	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185
100	0	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185
100	0	0,7742215	1,148241	1,947981	5,107681	2,843709	0,8313545	1,519726
100	0	9,024043e-16	1,010218e-15	1,703041e-15	8,157759e-16	1,332929e-15	1,11914e-15	1,563914e-15
100	0	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185
100	0	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185
100	0	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185	6,283185
100	0	1,085755	1,613739	2,980651	4,972976	3,606308	0,6875429	2,489145
100	0	6,035822e-15	5,764986e-15	9,391871e-15	6,283185	4,601921e-15	5,371261e-15	8,890606e-15
100	0	6,283185	6,283185	6,283185	2,248231e-15	6,283185	6,283185	6,283185
100	0	7,50973e-15	4,895275e-15	8,420039e-15	6,283185	6,283185	6,101004e-15	9,56792e-15
100	0	4,095285e-15	4,466331e-15	6,935234e-15	6,283185	4,906239e-15	3,958816e-15	6,336197e-15
100	0	9,005454e-16	1,011865e-15	1,704152e-15	8,108106e-16	1,338943e-15	1,117993e-15	1,563258e-15
100	0	0,9579977	0,8454016	1,729876	5,891264	0,6696078	4,139472	2,782176
100	0	5,195515	5,378406	3,566091	0,5776515	5,782584	1,836276	2,700585
100	0	9,273564e-17	1,674152e-16	3,731395e-16	1,233485e-16	2,991039e-16	1,168475e-16	3,094867e-16
100	0	0,7742215	1,148241	1,947981	5,107681	2,843709	0,8313545	1,519726
100	0	6,283185	6,283185	6,283185	1,346695e-15	6,283185	6,283185	6,283185
100	0	6,283185	6,283185	6,283185	1,049936e-13	6,283185	6,283185	6,283185

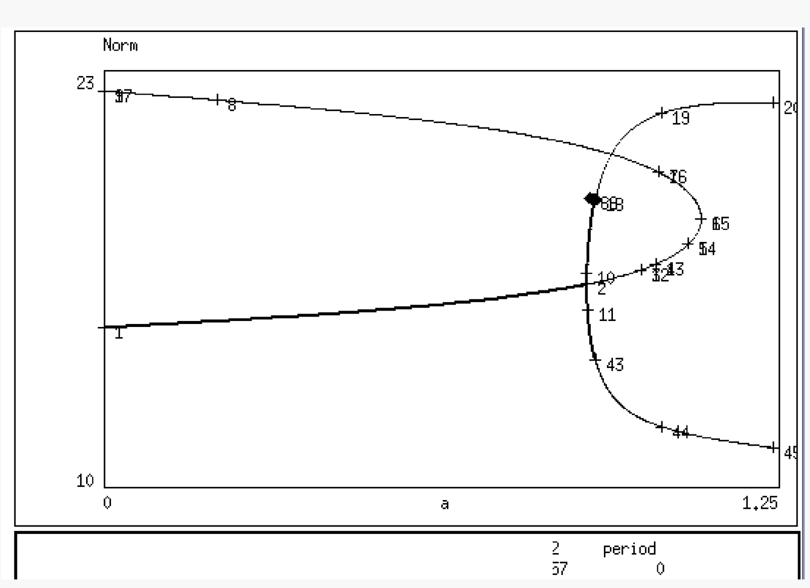
Figure 3. Oscillator Data

Results

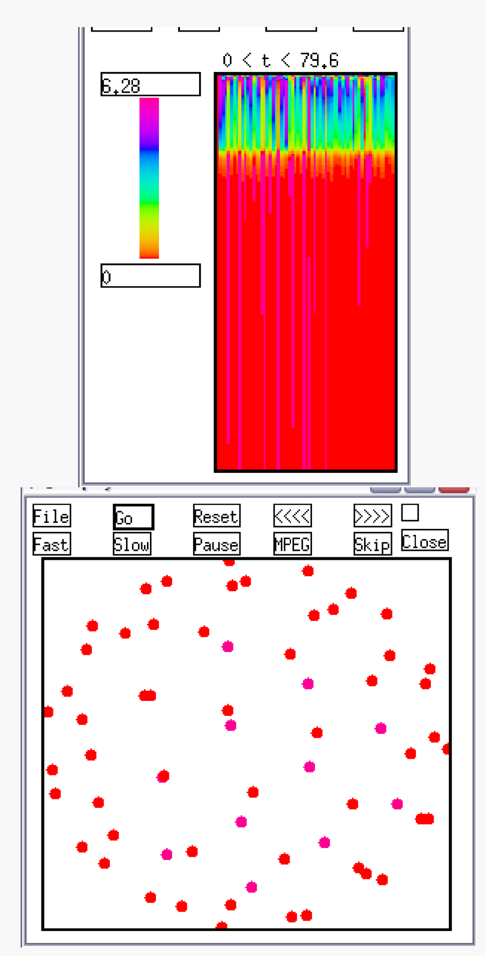
Dodecahedron Model: Non-synchronous movement found ~2% of the time with exactly three neighbors for each oscillator

N=64 Model: Non-synchronous patterns found ~10% of the time, with 5-7 neighbors for each oscillator

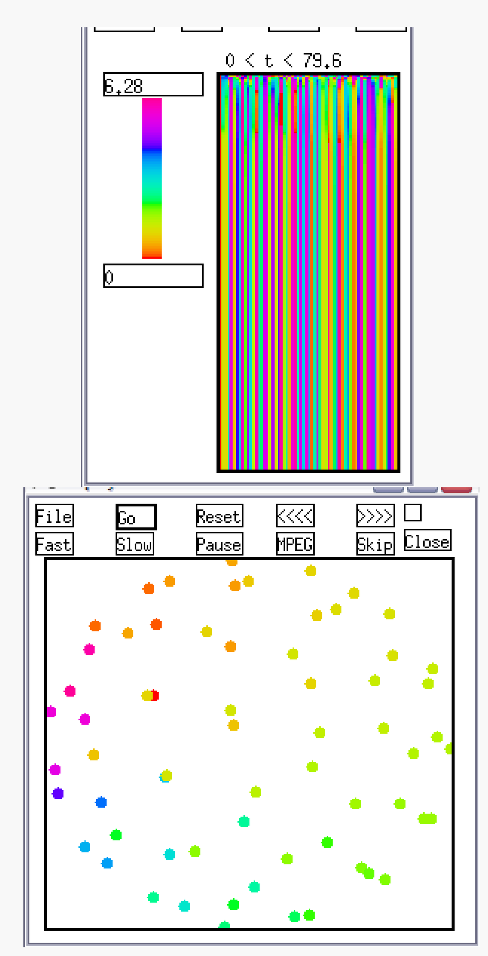
Sets of solutions were also found for the periodic function sin(x+a), and it was observed that solutions of increasing a became unstable after approximately a=1



Graph 1. Bifurcation Graph of sin(x+a)



Figures 4 and 5. Synchronous Movement



Figures 5 and 6. Non-synchronous Movement

Future Work

To develop a more detailed model of oscillating waves about a sphere, future research will focus on:

- Observing higher numbers of n oscillators on the surface
- Utilizing different periodic functions other than sin(x)
- Seeing not only oscillators but excitable or other active elements
- Altering the connectivity of networks to see if this affects the probability of synchronization
- Addition of long range connections with delays to see what this does (Such connections are common in the brain)

Acknowledgements

The national BBSI program (http://bbsi.eeicom.com) is a joint initiative of the NIH-NIBIB and NSF-EEC, and the BBSI @ Pitt is supported by the National Science Foundation under Grant EEC-0234002.

1) G. Bard Ermentout, Ph.D.
2) University of Pittsburgh

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