A Population Firing Rate Model of Reverberatory Activity in Neuronal Networks

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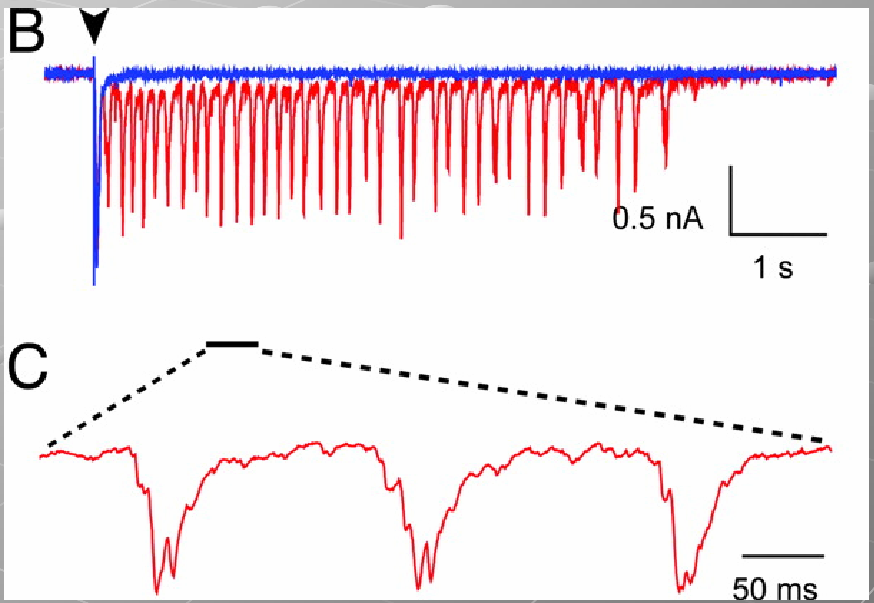
Reverberatory Activity

- Neural networks are able to maintain persistent activity after temporary inputs
- Input must have sufficient strength
- If so, network will go into long-term stable oscillation



Lau, Pak-Ming, and Guo-Qiang Bi. "Synaptic Mechanisms of Persistent Reverberatory Activity in Neuronal Networks." (2005)

Fast and Slow Synaptic Activity

- Fast Activity
  - Regulated by neurotransmitter signals
  - Result of action potential in adjacent neuron
  - Come in short bursts and cause depolarization that is quickly reverted back to resting state
  - ~ 5ms
- Slow Activity
  - Regulated by calcium ions
  - Elevates potential of membrane slightly
  - Takes much longer to return to resting state
  - ~200 ms

Spike Frequency Adaptation and Synaptic Depression

- Spike Frequency Adaptation
  - Gradual reduction of firing frequency
  - Implicated in habituation
- Synaptic Depression
  - As a cell fires continuously, it does not have enough time after firing to fully recover neurotransmitter level
  - Signals it releases become progressively weaker

Goals

- Create firing rate models of reverberatory activity in neurons incorporating fast synaptic activity, slow synaptic activity, and spike frequency adaptation or synaptic depression
- Analyze models for accuracy and robustness
- Create networks of coupled neurons based on the firing rate models

Deriving a Population Firing Rate Model

- Total response of post-synaptic cell at time t

$$\alpha(t-t_1) + \dots + \alpha(t-t_n) = \sum_{j=1}^{n} \alpha(t-t_j)$$

- The total response of all cells becomes

$$I(t) = \int_0^t \alpha(t-s)\left(\sum_{j=1}^{n} pr(s=t_j)\right)ds$$

- Let $\mu(t)$ equal the firing rate
- In a population model, the probability equals the firing rate

$$I(t) = \int_0^t \alpha(t-s)\mu(s)ds$$

Deriving a Population Firing Rate Model

- The firing rate of the post-synaptic cell is determined by the firing patterns of the pre-synaptic cell

$$\mu_{post}(t) = F(I_{post}(t)) = F(\int_0^t \alpha(t-s)\mu_{pre}(s)ds$$

- F is some nonlinear function of inputs
- In population models, $\mu_{pre}=\mu_{post}$, so

$$\mu(t) = F\left(\int_0^t \alpha(t-s)\mu(s)ds\right)$$

Spike Frequency Adaptation Model

$$F(x) = \sqrt{\frac{x}{1 - e^{-bx}}}$$

$$\mu(t) = \sqrt{\frac{(gf * sf + gs * ss - ga * sa - ithr)}{1 - e^{-b(gf * sf + gs * ss - ga * sa - ithr)}}}$$

$$sf' = \frac{-sf(t) + \mu(t)}{tf}$$

$$ss' = \frac{-ss(t) + \mu(t)(1 - ss(t))}{ts}$$

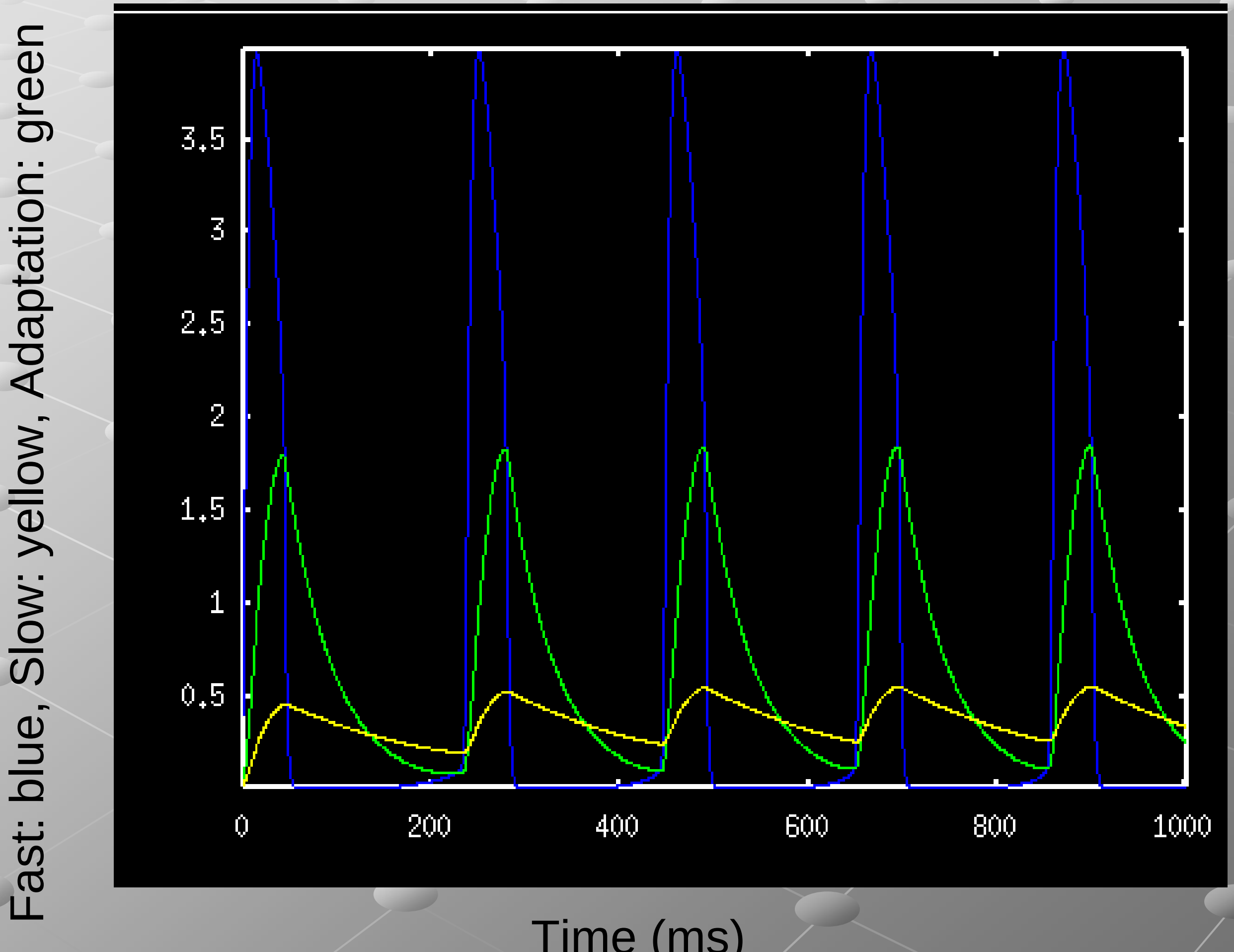
$$sa' = \frac{-sa(t) + \mu(t)}{ta}$$

Parameters

- tf=2
- ts=200
- ta=50
- gf=5
- gs=2
- ga=4
- b=8
- ithr=1

Spike Frequency Adaptation Model

- Model showed stable, long-term oscillation

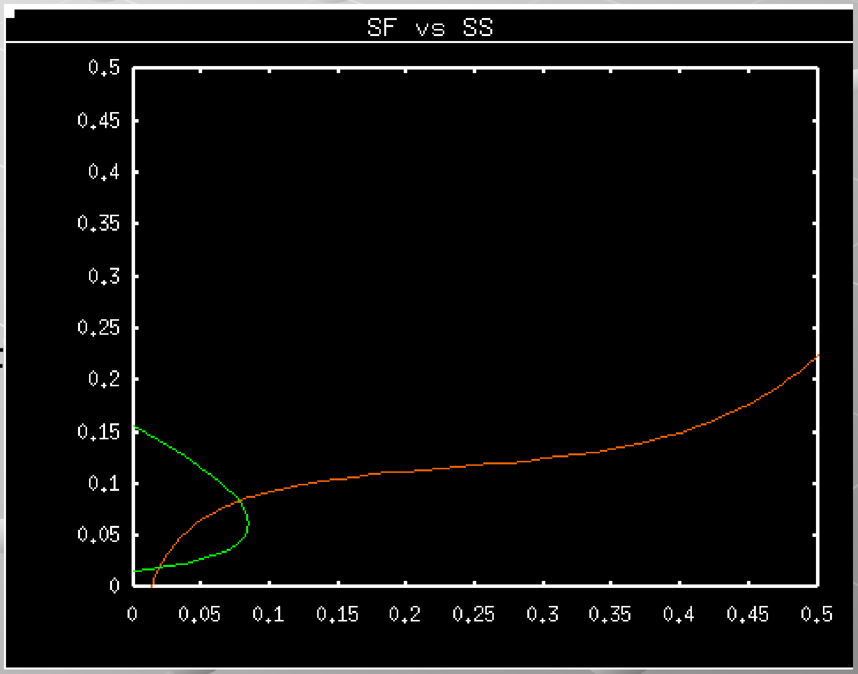


Spike Frequency Adaptation Model

- Nullclines show two equilibrium points, one stable and one unstable

Nullclines

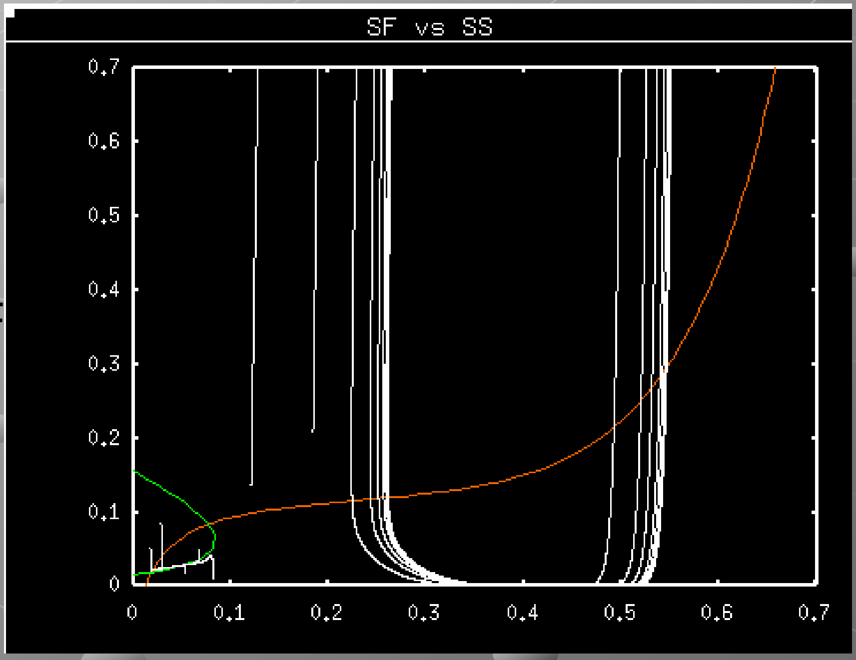
SF



SS

Integrated

SF



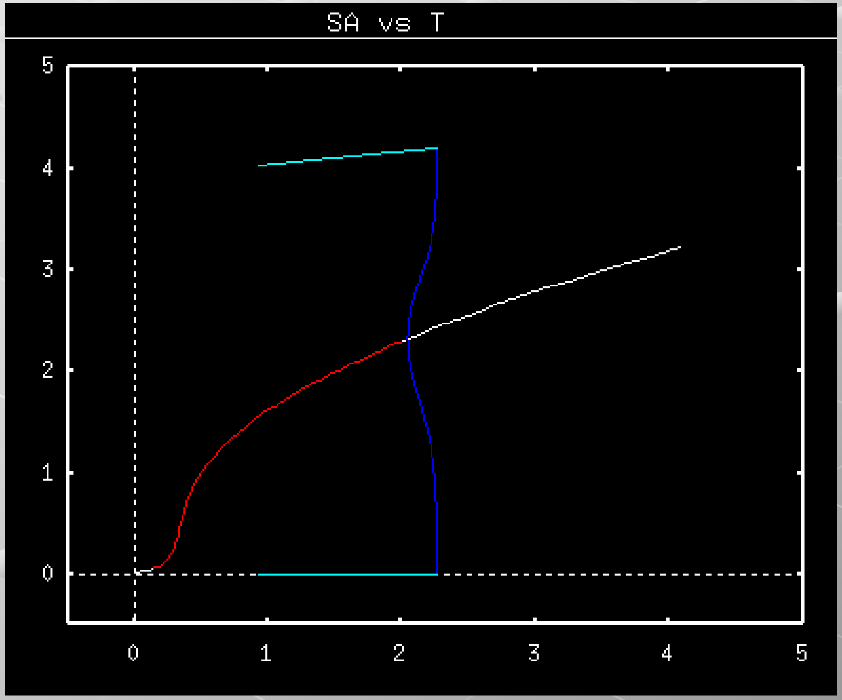
SS

Analysis: Spike Frequency Adaptation Model

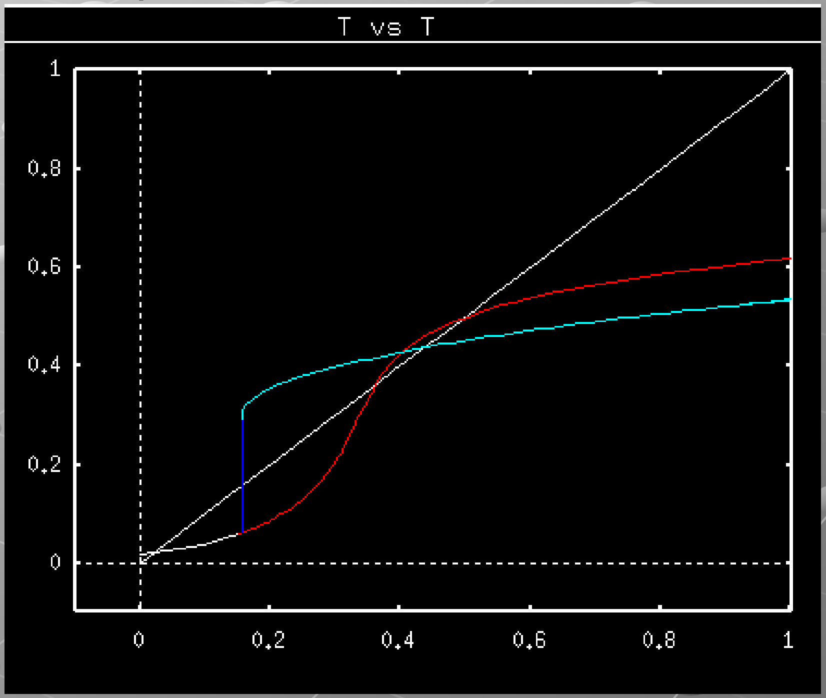
- Need to make sure oscillation is robust, and not a fluke occurrence
- Treat SS as a parameter, conduct bifurcation analysis
- SS as parameter should be same as average of SS in oscillations

Analysis: Spike Frequency Adaptation Model

Bifurcation diagram with extended Hopf bifurcation



Intersection of average value of SS the parameter and SS the variable



- Intersection occurs at .438

Synaptic Depression Model

$$F(x) = \sqrt{\frac{x}{1 - e^{-bx}}}$$

$$\mu(t) = \sqrt{\frac{(gf * sf + gs * ss) * q - ithr}{1 - e^{-b((gf * sf + gs * ss) * q - ithr)}}}$$

$$sf' = \frac{-sf(t) + \mu(t)}{tf}$$

$$ss' = \frac{-ss(t) + as * \mu(t)^{p} * (1 - ss(t))}{ts}$$

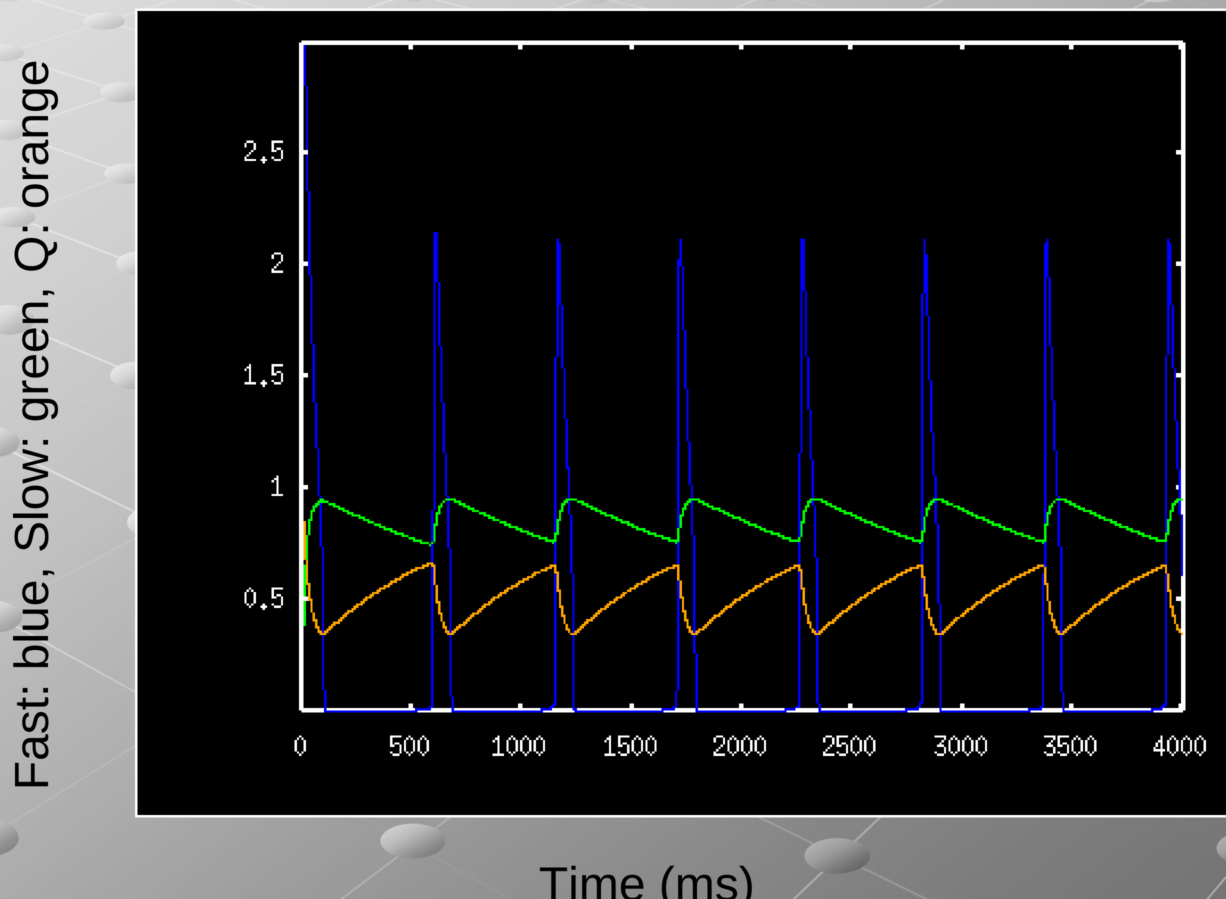
$$q' = \frac{1 - q(t) - alpha * q(t) * \mu(t)}{tq}$$

Parameters

- tf=2
- ts=2000
- gf=4
- gs=2.2
- tq=700
- alpha=5
- b=40
- ithr=1.3

Synaptic Depression Model

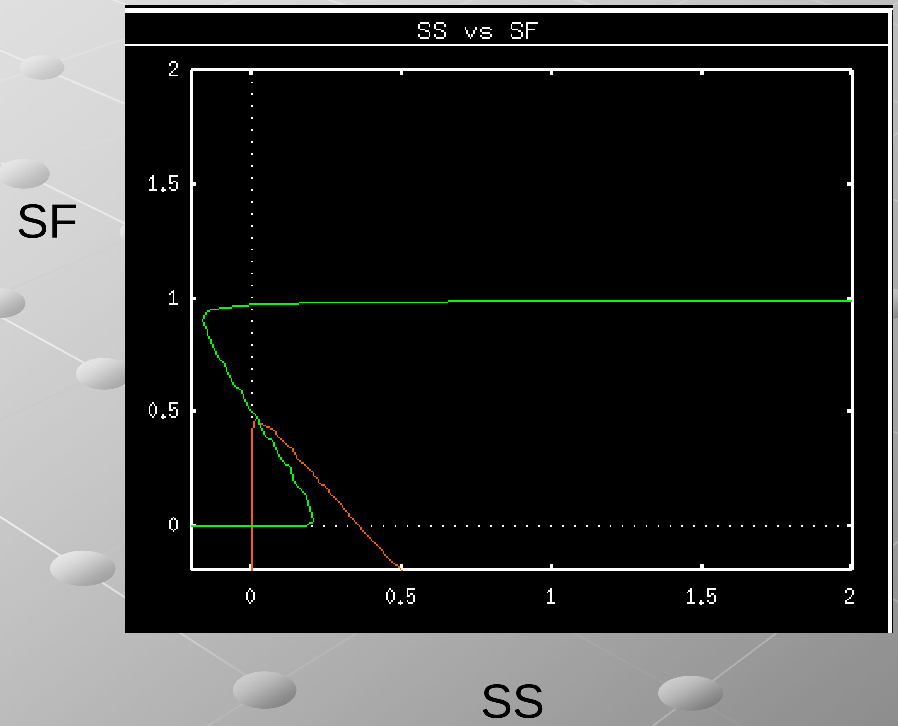
- Model showed stable, long-term oscillation



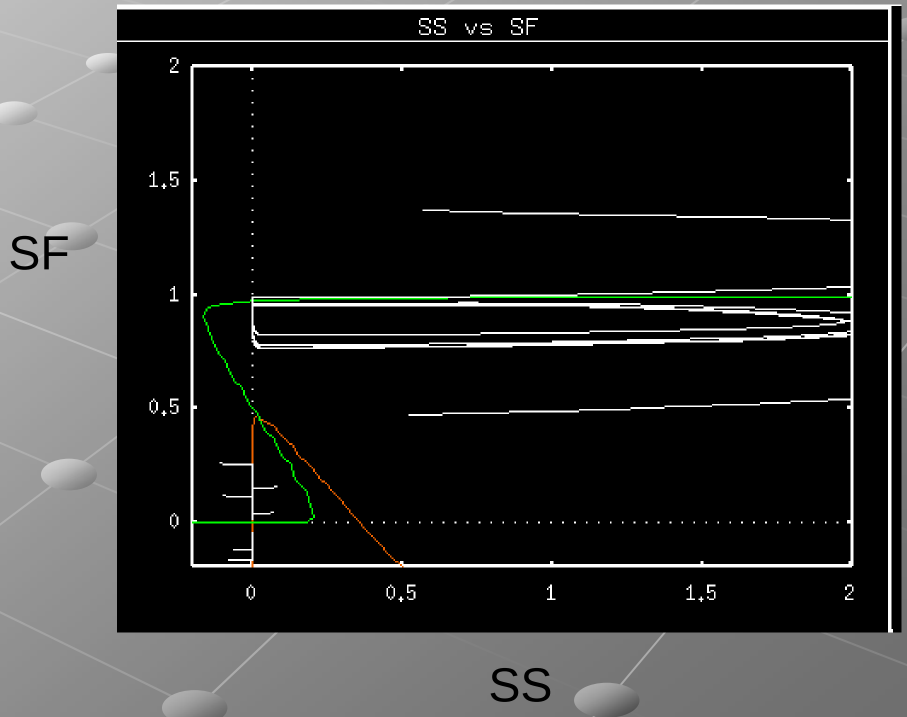
Synaptic Depression Model

- Nullclines show two equilibrium points, one stable and one unstable

Nullclines

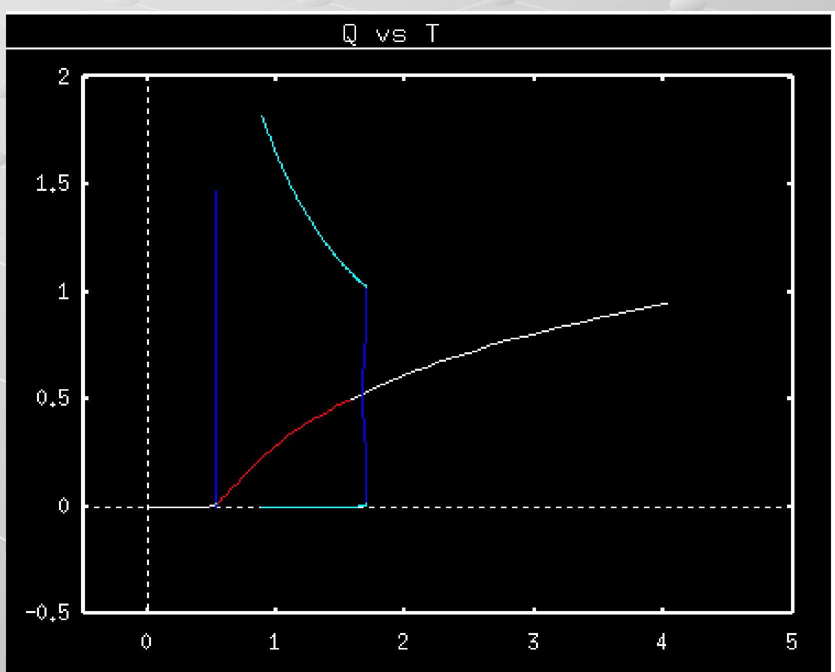


Integrated



Analysis: Synaptic Depression Model

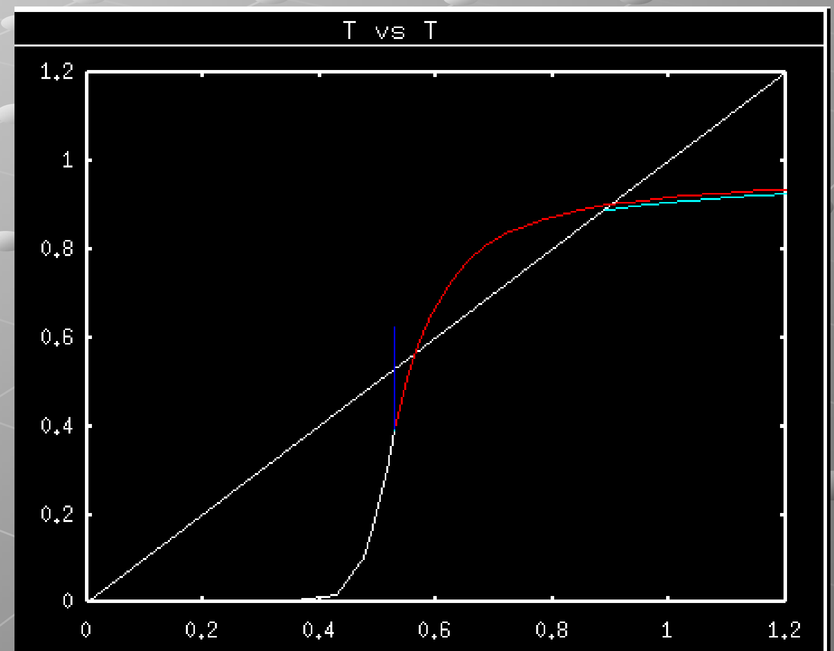
Bifurcation diagram with extended Hopf bifurcation



T

Intersection of average value of SS the parameter and SS the variable

SS (variable)



SS (parameter)

- Intersection occurs at .890

Spike Frequency Adaptation Network

The Coupling Terms

$$sf[1] = \frac{sf[1] + beta * sf[2]}{1 + beta}$$

$$sf[2...19] = \frac{sf[j] + beta * (sf[j+1] + sf[j-1])}{1 + 2 * beta}$$

$$sf[20] = \frac{sf[20] + beta * sf[19]}{1 + beta}$$

$$ss[1] = \frac{ss[1] + beta * ss[2]}{1 + beta}$$

$$ss[2...19] = \frac{ss[j] + beta * (ss[j+1] + ss[j-1])}{1 + 2 * beta}$$

$$ss[20] = \frac{ss[20] + beta * ss[19]}{1 + beta}$$

Spike Frequency Adaptation Network

Adaptation of Uncoupled Model

$$F(x) = \sqrt{\frac{x}{1-e^{-bx}}}$$

$$\mu[1..20] = \sqrt{\frac{gf * sf[j] + gs * ss[j] - ga * sa[j] - ithr}{1-e^{-b(gf*sf[j]+gs*ss[j]-ga*sa[j]-ithr)}}}$$

$$sf[1..20]' = \frac{-sf[j] + \mu[j]}{tf}$$

$$ss[1..20]' = \frac{-ss[j] + \mu[j]*(1-ss[j])}{ts}$$

$$sa[1..20]' = \frac{-sa[j] + \mu[j]}{ta}$$

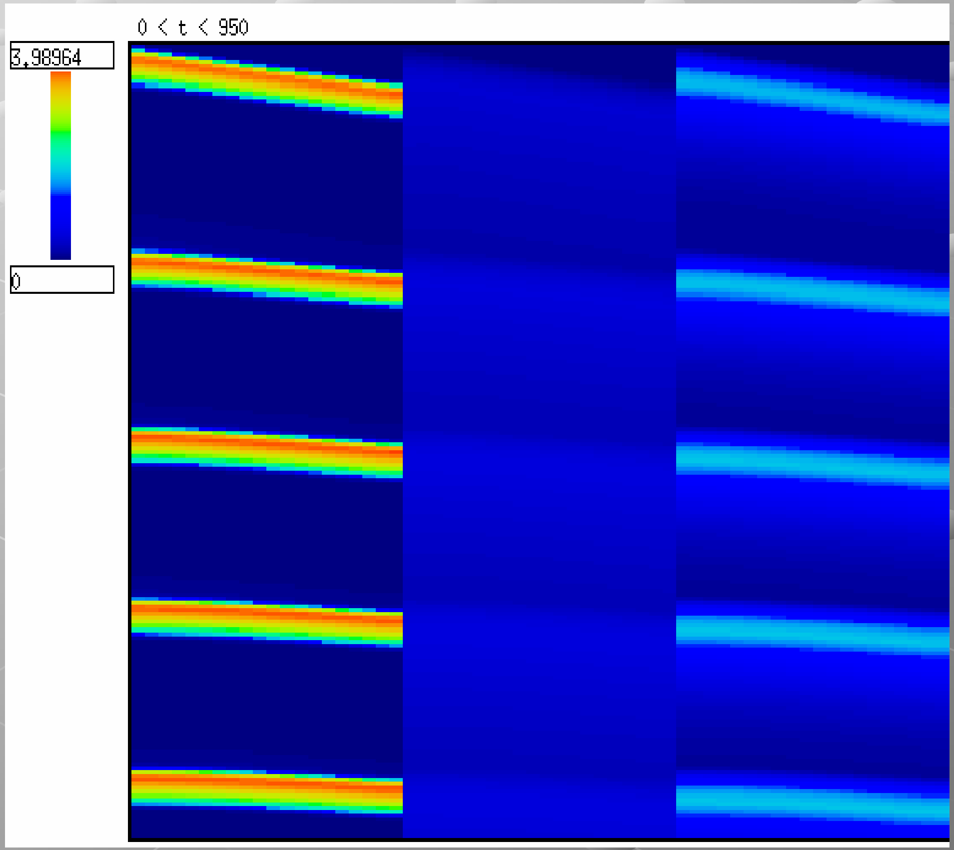
Parameters

- tf=2
- ts=200
- ta=50
- gf=5
- gs=2
- ga=4
- b=8
- ithr=1

Spike Frequency Adaptation Network

- Initial conditions: sf[1]=.4, the rest 0

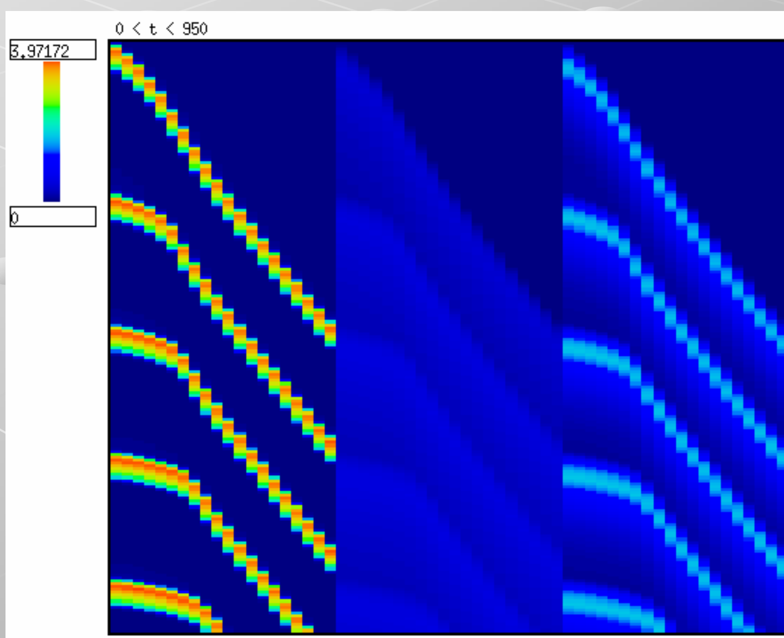
SF SS SA



Spike Frequency Adaptation Network

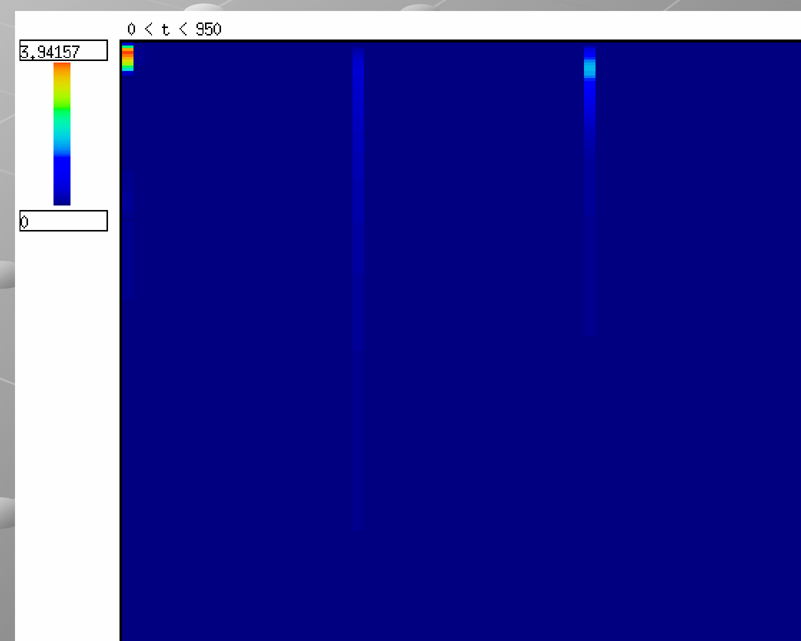
- Synchronization was observed even with very small beta

SF SS SA



beta=.009

SF SS SA



beta=.006

Synaptic Depression Network

- Coupling terms same

$$F(x) = \sqrt{\frac{x}{1-e^{-bx}}}$$

$$\mu[1..20] = \sqrt{\frac{(gf*sf[j]+gs*ss[j])*q[j]-ithr}{1-e^{-b((gf*sf[j]+gs*ss[j])*q[j]-ithr)}}}$$

$$sf[1..20]' = \frac{-sf[j]+\mu[j]}{tf}$$

$$ss[1..20]' = \frac{-ss[j]+as*\mu[j]^{p}*(1-ss[j])}{ts}$$

$$q[1..20]' = \frac{1-q[j]-alpha*q[j]*\mu[j]}{tq}$$

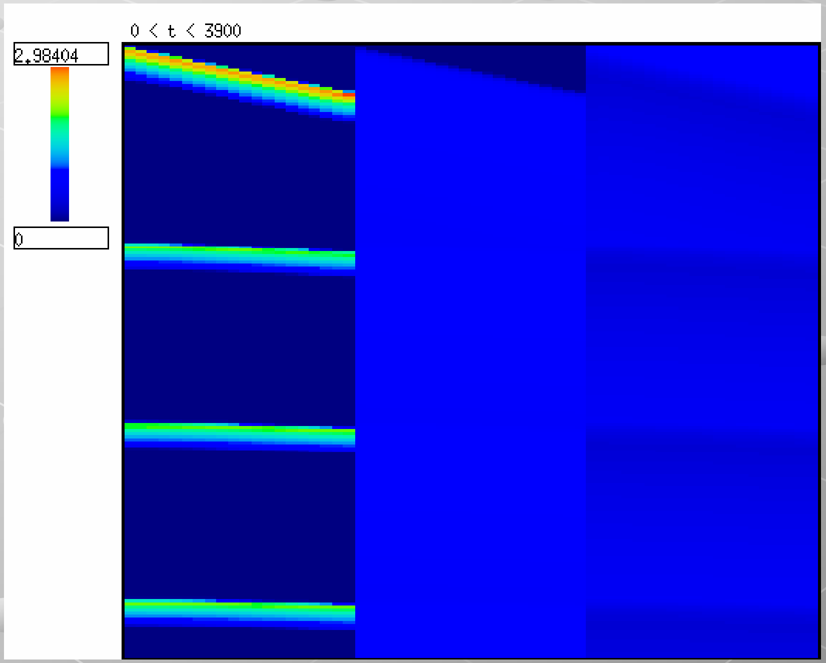
Parameters

- beta=.2
- tf=2
- ts=2000
- gf=4
- gs=2.2
- tq=700
- alpha=5
- b=40
- ithr=1.3
- as=40
- p=1

Synaptic Depression Network

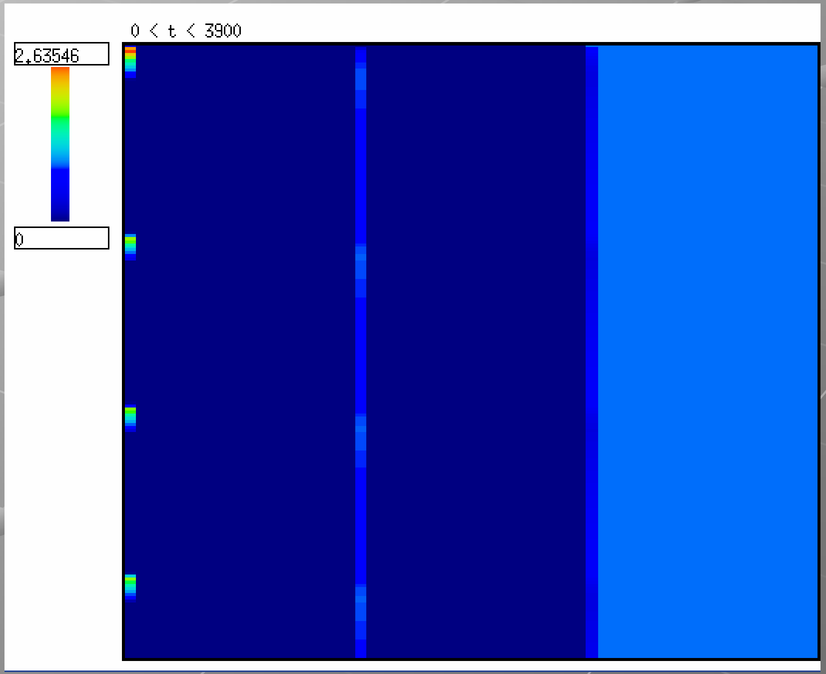
- Initial conditions: sf[1]=.5, q[1..20]=1, rest 0

SF SS SA



beta=.2

SF SS SA



beta=.1

Conclusions

- Neural activity incorporating fast synaptic activity, slow synaptic activity, spike frequency adaptation and synaptic depression can be modeled with population firing rate models
- These models are accurate and robust
- Network models simulate the activity of multiple systems incorporating these factors

References

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- Benda, Jan, Longtin Andre, and Len Maler. "Spike-Frequency Adaptation Separates Transient Communication Signals From Background Oscillations." The Journal of Neuroscience 25 (2005): 2312-2321.
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Questions?