A Population Firing Rate Model of Reverberatory Activity in Neuronal Networks

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Abstract

Synaptic activity based on neurotransmitters has been thoroughly described by mathematical models. Most of the existing models, however, disregard asynchronous synaptic transmission, another type of synaptic signal. Asynchronous synaptic transmission has been shown to elevate the potential of the cell slightly, and take a much longer time to return to resting state then the regular neurotransmitter signal. In this presentation, we will discuss our results in 1) creating firing rate models of neuronal networks incorporating asynchronous synaptic transmission as well as spike frequency adaptation or synaptic depression 2) creating network models with all neurons affecting each other based on the firing rate models 3) analyzing all models. These results emphasize the impact of asynchronous synaptic transmission on neural activity.

Introduction

Asynchronous synaptic transmission is regulated by calcium ions, has been shown to elevate the potential of the cell slightly, and take a much longer time to return to resting state than the regular a neurotransmitter signal. This signal raises the potential of the cell more than the regular signal would by itself, and could cause the cell to reach the depolarization threshold and fire an action potential that it would not have otherwise. In my research, I explore asynchronous synaptic transmission and incorporate two characteristics of neuronal networks, spike frequency adaptation and synaptic depression.

A population firing rate model is based on determining a function for the potential of a post-synaptic cell. This function is characterized a low, flat curve, with interspaced spikes representing the action potentials. The total response of the system is.

$$I(t) = \int_0^t \alpha(t-s)\mu(s)ds$$

Where α is the potential of the post-synaptic cell and μ is the firing rate. In any particular synapse, the firing rate of the post-synaptic cell is determined by the firing patterns of the pre-synaptic cell. Thus,

$$\mu_{post}(t) = F(I_{post}(t)) = F\left(\int_0^t \alpha(t-s)\mu_{pre}(s)ds\right)$$

This means that the firing rate of a post-synaptic cell is a function of the response of that cell where F is some nonlinear function of inputs. One of the fundamental assumptions of a population model is that the firing rates of all neurons are the same, thus

$$\mu(t) = F\left(\int_0^t \alpha(t-s)\mu(s)ds\right)$$

All population models use this basic problem structure, but vary in the selection of the α function.

Methods

•Equations for population model with spike frequency adaptation

$$F(x) = \sqrt{\frac{x}{1-e^{-bx}}}$$

$$\mu(t) = \sqrt{\frac{(gf*sf + gs*ss - ga*sa - ithr)}{1-e^{-b(gf*sf+gs*ss-ga*sa-ithr)}}}$$

$$sf' = \frac{-sf(t) + \mu(t)}{tf}$$

$$ss' = \frac{-ss(t) + \mu(t)(1-ss(t))}{ts}$$

$$sa' = \frac{-sa(t) + \mu(t)}{ta}$$

•Equations for population model with synaptic depression

$$F(x) = \sqrt{\frac{x}{1-e^{-bx}}}$$

$$\mu(t) = \sqrt{\frac{(gf*sf + gs*ss)*q - ithr}{1-e^{-b((gf*sf+gs*ss)*q-ithr)}}}$$

$$sf' = \frac{-sf(t) + \mu(t)}{tf}$$

$$ss' = \frac{-ss(t) + as*\mu(t)^p*(1-ss(t))}{ts}$$

$$q' = \frac{1-q(t) - alpha*q(t)*\mu(t)}{tq}$$

•Coupling terms for both network models

$$sf[1] = \frac{sf[1] + beta*sf[2]}{1+beta}$$

$$sf[2...19] = \frac{sf[j] + beta*(sf[j+1]+sf[j-1])}{1+2*beta}$$

$$sf[20] = \frac{sf[20] + beta*sf[19]}{1+beta}$$

$$ss[1] = \frac{ss[1] + beta*ss[2]}{1+beta}$$

$$ss[2...19] = \frac{ss[j] + beta*(ss[j+1]+ss[j-1])}{1+2*beta}$$

$$ss[20] = \frac{ss[20] + beta*ss[19]}{1+beta}$$

•Network adaptation of the spike frequency adaptation model

$$F(x) = \sqrt{\frac{x}{1-e^{-bx}}}$$

$$\mu[1..20] = \sqrt{\frac{gf*sf[j] + gs*ss[j] - ga*sa[j] - ithr}{1-e^{-b(gf*sf[j]+gs*ss[j]-ga*sa[j]-ithr)}}}$$

$$sf[1..20]' = \frac{-sf[j] + \mu[j]}{tf}$$

$$ss[1..20]' = \frac{-ss[j] + \mu[j]*(1-ss[j])}{ts}$$

$$sa[1..20]' = \frac{-sa[j] + \mu[j]}{ta}$$

•Network adaptation of the synaptic depression model

$$F(x) = \sqrt{\frac{x}{1-e^{-bx}}}$$

$$\mu[1..20] = \sqrt{\frac{(gf*sf[j] + gs*ss[j])*q[j] - ithr}{1-e^{-b((gf*sf[j]+gs*ss[j])*q[j]-ithr)}}}$$

$$sf[1..20]' = \frac{-sf[j] + \mu[j]}{tf}$$

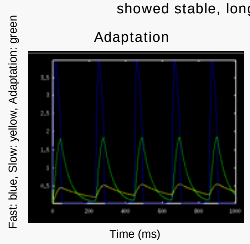
$$ss[1..20]' = \frac{-ss[j] + as*\mu[j]^p*(1-ss[j])}{ts}$$

$$q[1..20]' = \frac{1-q[j] - alpha*q[j]*\mu[j]}{tq}$$

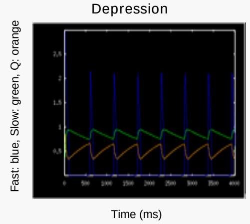
Results

Adaptation and depression models showed stable, long term oscillation

Adaptation

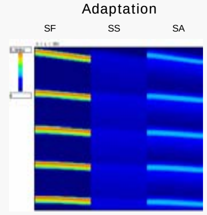


Depression

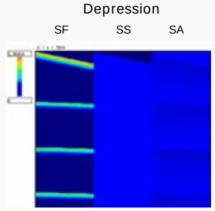


Network models also showed long tem oscillation

Adaptation

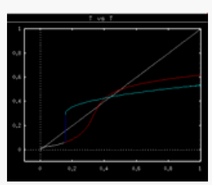


Depression

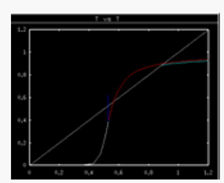


Analysis of asynchronous synaptic transmission variable revealed the oscillation to be accurate

Adaptation



Depression



The intersection of average value of parameter SS and variable SS

Conclusions

- •Neural activity incorporating fast synaptic activity, slow synaptic activity, spike frequency adaptation and synaptic depression can be modeled with population firing rate models
- •These models are accurate and robust
- •Network models simulate the activity of multiple systems incorporating these factors

Future Work

- •Expand on current models by incorporating other kinds of neural activity
- • Expand in network models by making them larger, and coupling several networks together

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