# *Beyond a pacemaker's entrainment limit: phase walk-through*

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Based on a paper by G. Bard Ermentrout and John Rinzel
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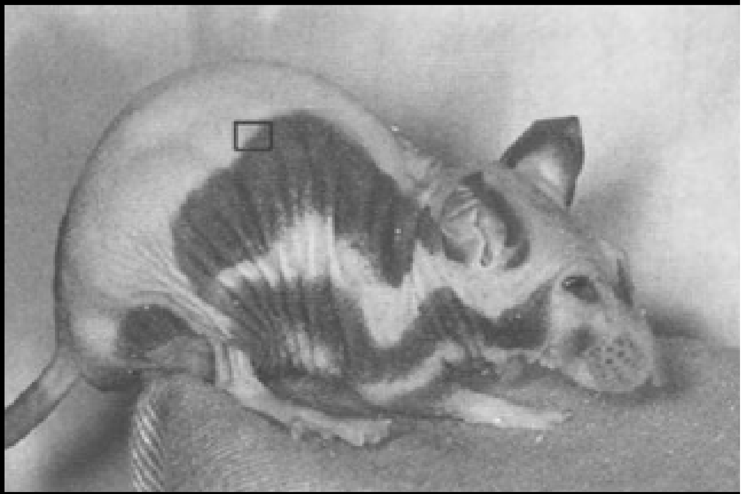
# *Fireflies*

"A great belt of light, some ten feet wide, formed by thousands upon thousands of fireflies whose green phosphorescence bridges the shoulder-high grass. . . The fluorescent band composed of these tiny organisms lights up and goes out with a precision that is perfectly synchronized."

- Joy Adamson

# *Examples of Biological Synchronization*

- Pacemaker cells in the heart
- Discharging of brain cells during epileptic seizures
- Women's menstrual cycles
- Hair growth in rodents

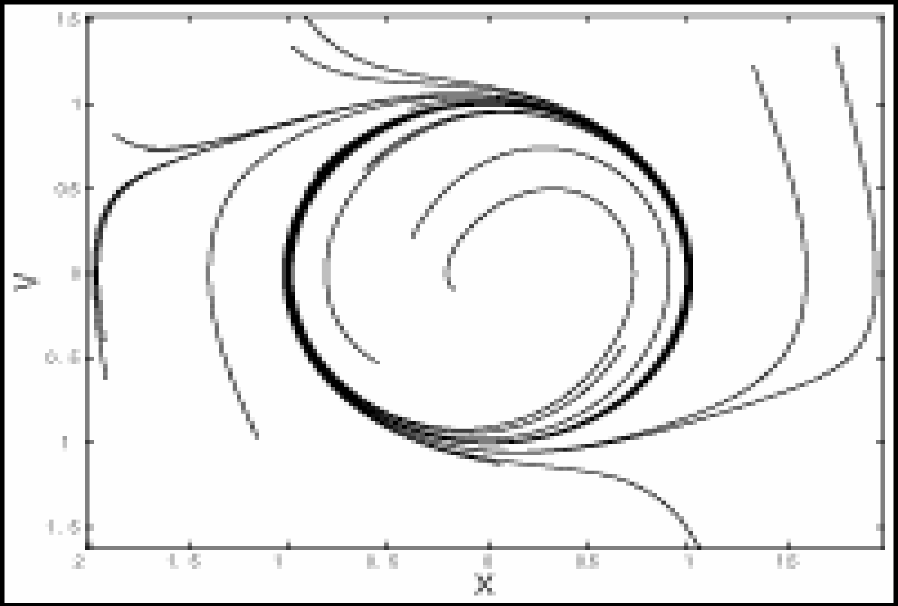


# Objectives

- Mathematically model the entrainment, loss of entrainment, and phase walk-through for coupled pace-makers
- Determine the relationship between frequency of the zeitgeber and the response of the oscillator

# *On the small scale*

- $T_0$ = natural period of oscillator
- T = period of stimulus
- $D\theta/dt = 2\pi/T_0$

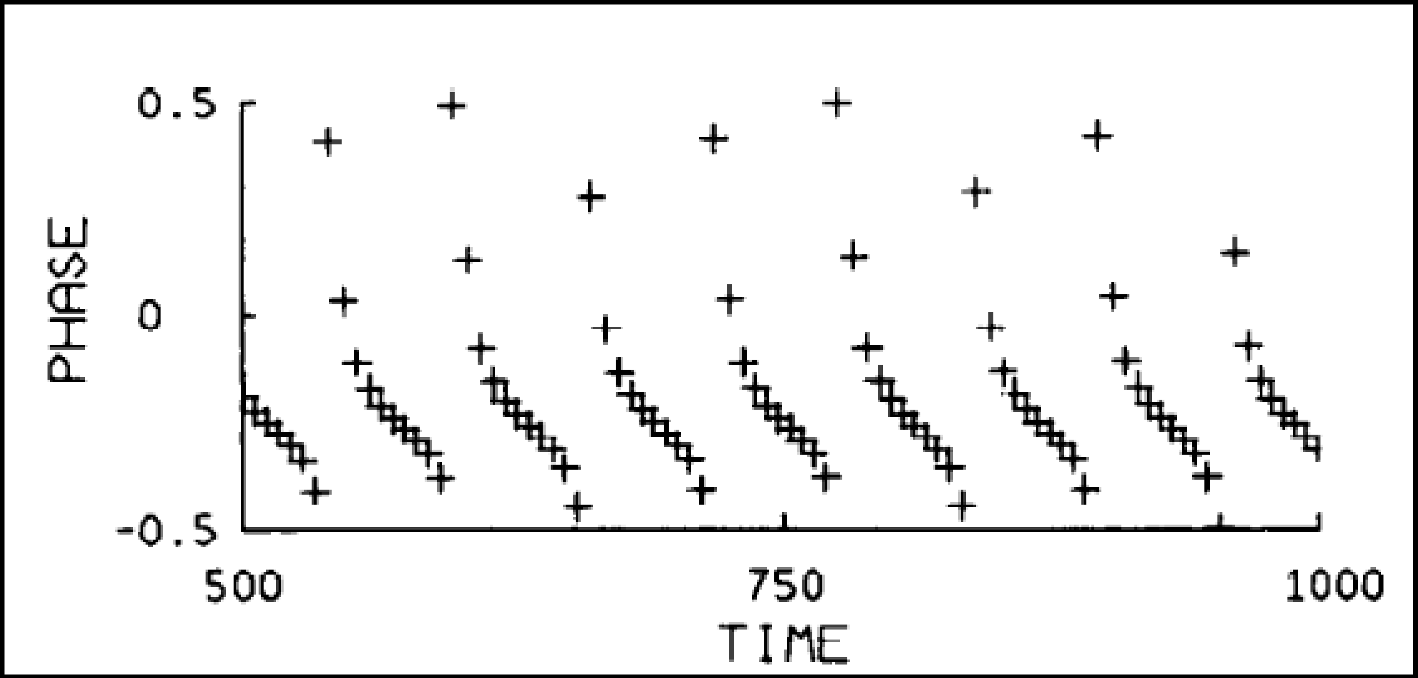


# *Entrainment and loss of entrainment*

- Depends on the difference between the natural frequency of the oscillator and the frequency of the zeitgeber
  - $T > T_e$ – synchronization
  - $T < T_e$ - desychronization
- Can synchronize out of phase

# *Phase walk-through*

- The phase difference between the oscillator and the stimulus varies periodically

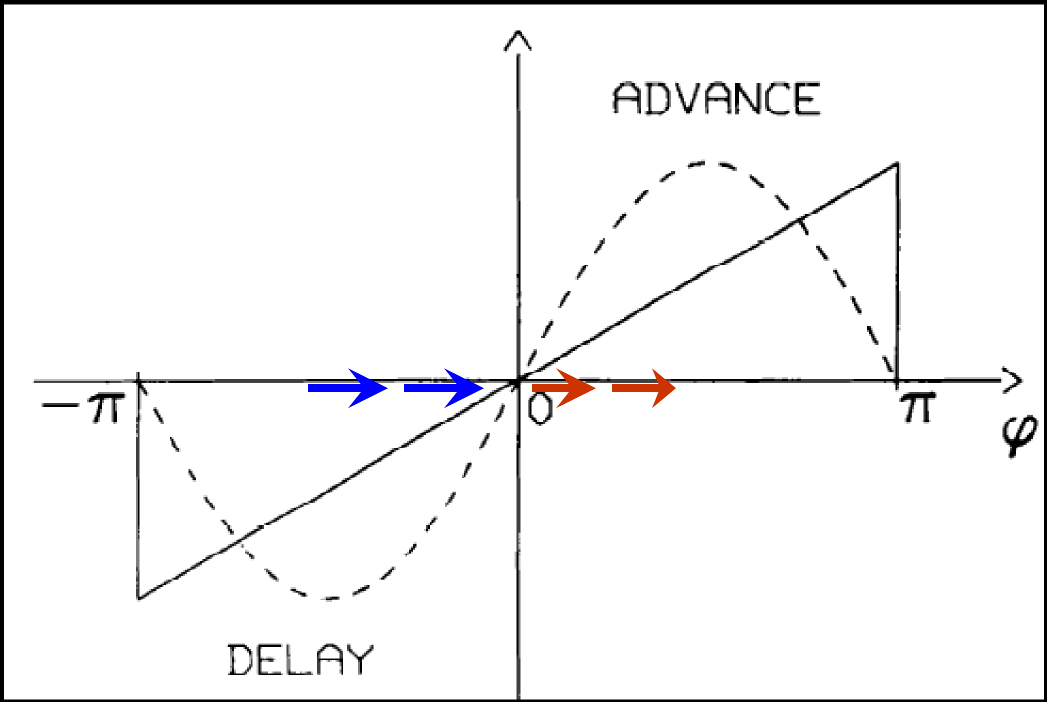


# Coupled frequency

$$\frac{d\theta}{dt} = 2\pi / T_0 + \beta \sin(\omega t - \theta)$$

$$\phi = \omega t - \theta$$

$$\omega = 2\pi / T$$



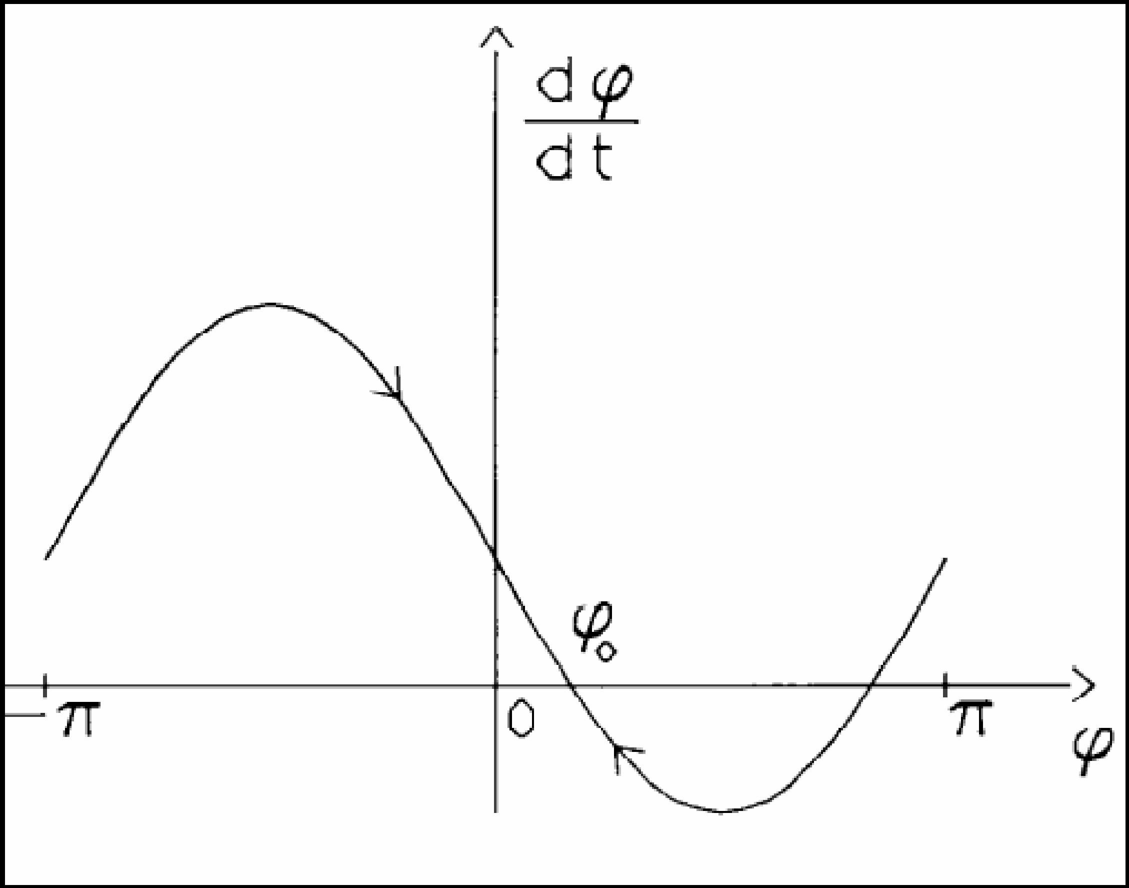
# On the large scale

$$\frac{d\theta}{dt} = \omega - \frac{d\phi}{dt}$$

$$\frac{d\phi}{dt} = \frac{2\pi}{T_0 T}(T_0 - T) - \beta \sin(\phi)$$

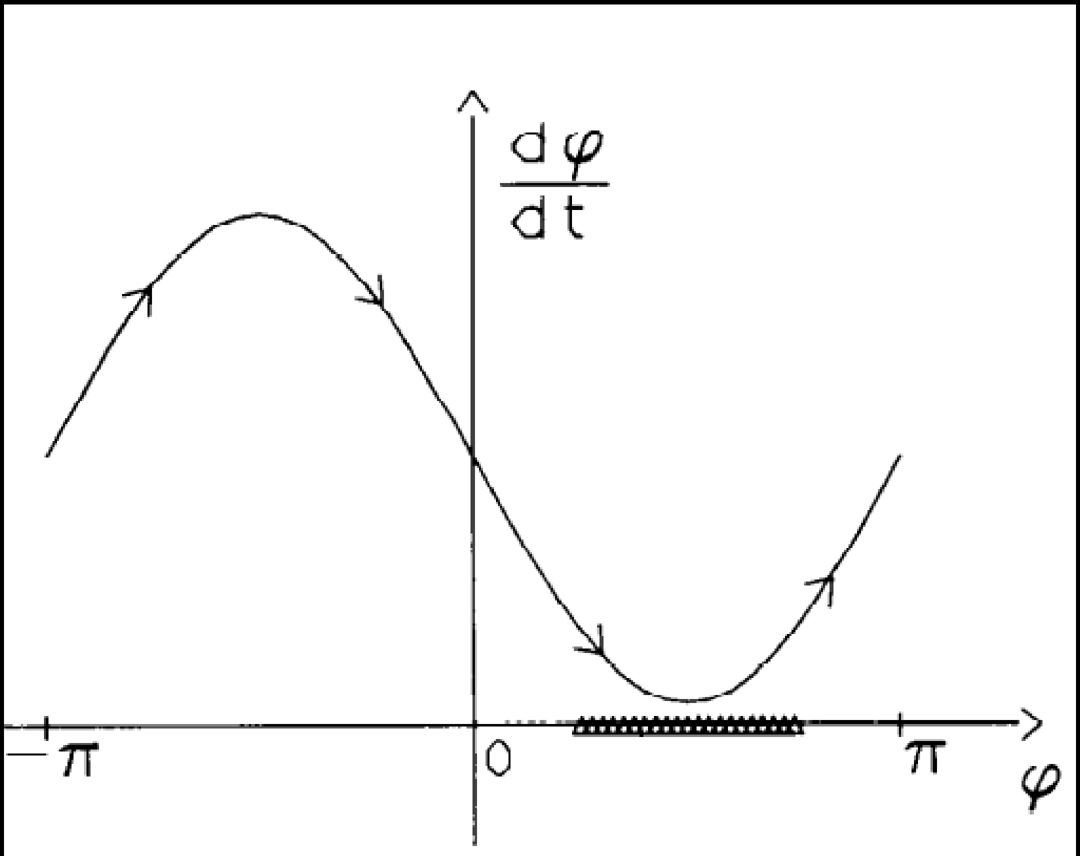
# *Change in phase difference*

$T_0$-T small

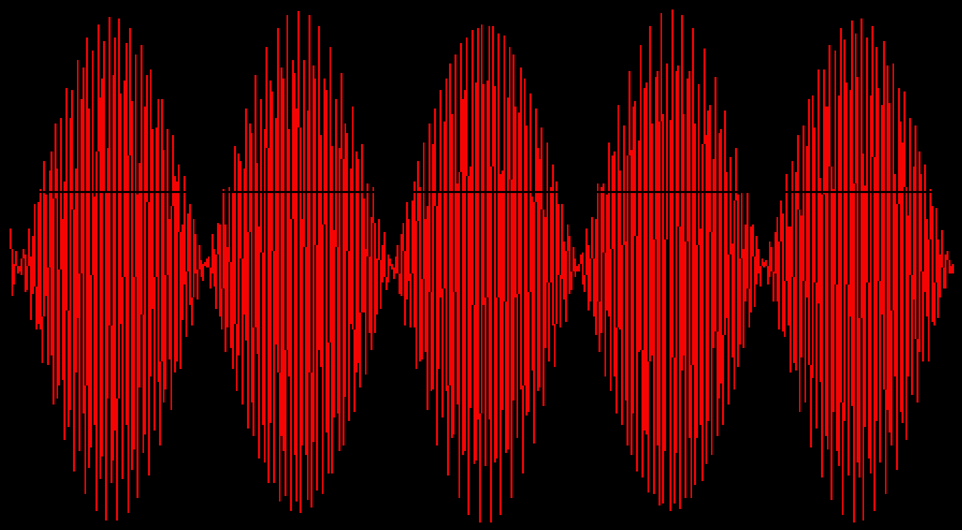


# Change in Phase Difference Cont

$T_0$-T large



# *Nested oscillations*



Beats

$$T_b = \frac{C(T_e, T_0)}{\sqrt{\frac{T_e}{T} - 1}}$$

# *Important Notes*

- Just because $\phi$ is periodic, $\theta$ does not necessarily have a period of $T_{beat}$
- Results apply to weakly driven oscillators
- Phase walk through can occur if T is slightly above or slightly below $T_e$.

# *Conclusions*

- Fireflies serve as a convenient model for coupled oscillators
- $T > T_e$, synchronization
- $T - T_e$ small, phase walk-through
- $T < T_e$, desynchronization

# *Possible Applications*

- Intestine – frequency plateaus
- Circadian rhythms
- Psychology
  - Bipolar disorder
  - SAD

# *References*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