Optimization by Simulated Annealing

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Combinatorial Optimization

- Central to science, engineering, CS
- Optimizing objective (cost) functions of complicated systems
- Determination of global extremum of objective functions
- Deterministic methods unrealistic as number of parameters becomes very large
  - Traveling Salesman Problem
  - Computer Design

Introduction to the Traveling Salesman Problem

- **Problem 1**: Minimize cost function (E) of a salesman traveling between N number of cities and back
  - Cost to travel between cities proportional to distance between cities

Simplest form :

$$E = L \equiv \sum_{i=1}^{N} \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$$

Intro. Traveling Salesman Problem

continued…

- Number of possible path configurations = N!
- Exact solutions to minimization of E computationally determined for magnitudes of $N <\sim 10^2$ (as of 1983)
- Non-deterministic polynomial time complete (NP-C) problem
  - Computing effort for exact soln. increases exponentially with N
- Heuristic methods for near-optimal solutions
  - "Divide and Conquer" and "Iterative Improvements"

    →Monte Carlo (MC) & Simulated Annealing (SA)

Metropolis Monte Carlo Algorithm

1) Start with known configuration, objective function (ie, energy), some Temperature value

2) Random change configuration (ie, add small random displacement)

3) Calculate new energy value (E_2)

4) Compare to energy at previous position (E_1):
   - If $E_2 < E_1$, keep new position
   - If $E_2 > E_1$, keep new position if the Boltzmann factor for transition is greater or equal to a random number between 0 and 1

$$\mathrm{Rand}(0,1) \leq \exp[-(E_2 - E_1)/kT]$$

5) Repeat steps 2) – 4) K number of times

Simulated Annealing (SA)

- Concept of SA from annealing process
  - Slowly cooling melt to form perfect crystals

- SA provides a temperature schedule for the Metropolis method
  - Start at effectively high temperature and gradually decrease the temperature by increments until T slightly above 0 (<1)
  - At every temperature, Metropolis algorithm is run (nested-loop)

- Benefits
  - Ability to escape local minima at non-zero temperatures
  - “Divide and Conquer” -> Gross features of final state appear at high temp. while fine details appear at low temp.

SA Application: The Traveling Salesman

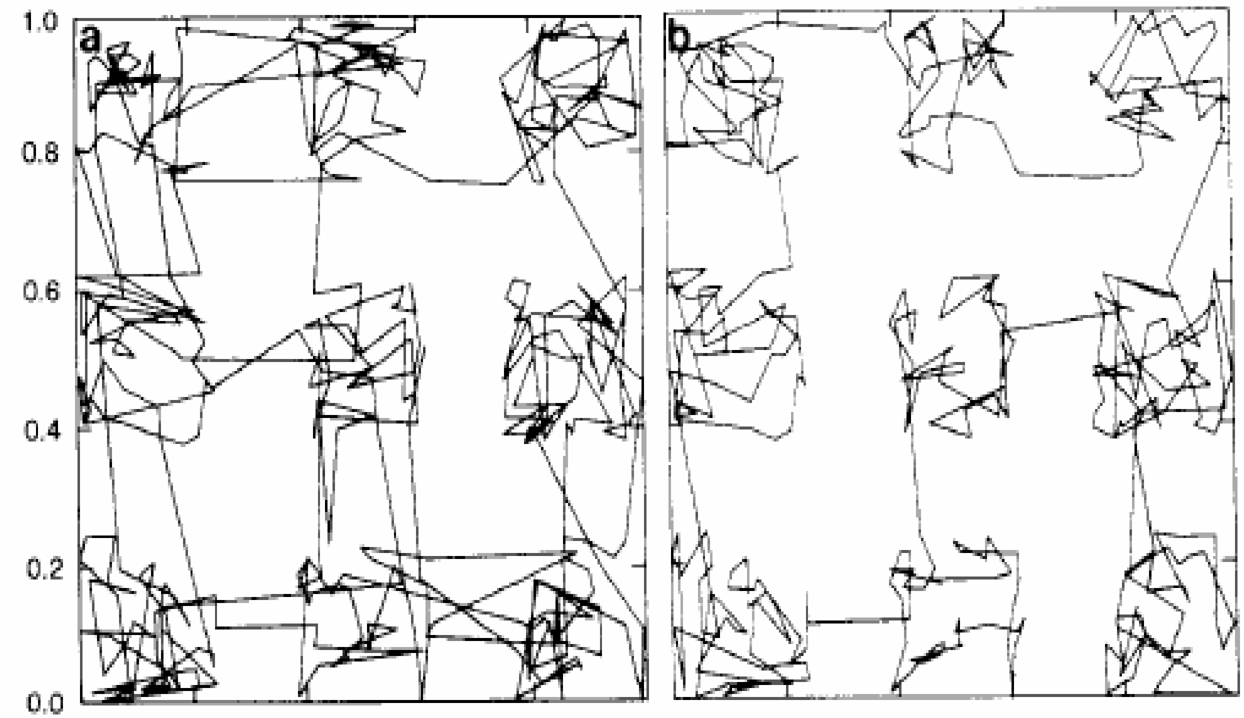
- Kirkpatrick et al solved problem where N=400
  - Re-arrangement involved random selection of string of cities and reversal of order (Lin-Kernighan method)
  - Side of square boundary has length of $N^{1/2}$
  - Cities grouped into nine clusters
  - Solved problem in "Manhattan" metric space so thus,

$$E = L \equiv \sum_{i=1}^{N} \left(\left| x_i - x_{i+1} \right| + \left| y_i - y_{i+1} \right| \right)$$

  - Solved problem several times and averaged optimal step lengths (α)

SA Results: Traveling Salesman

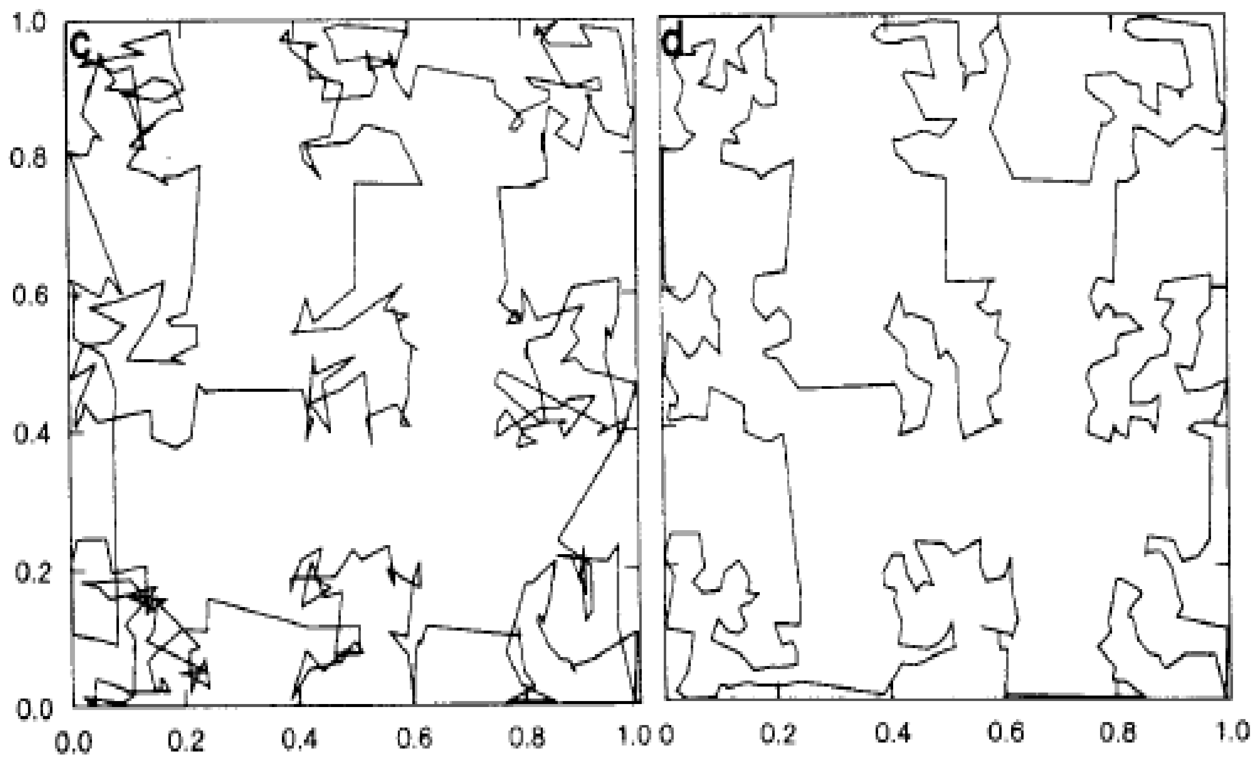
a) Results at T = 1.2 (α = 2.0567)
b) Results at T = 0.8 (α = 1.515)



SA Results: Traveling Salesman
continued…

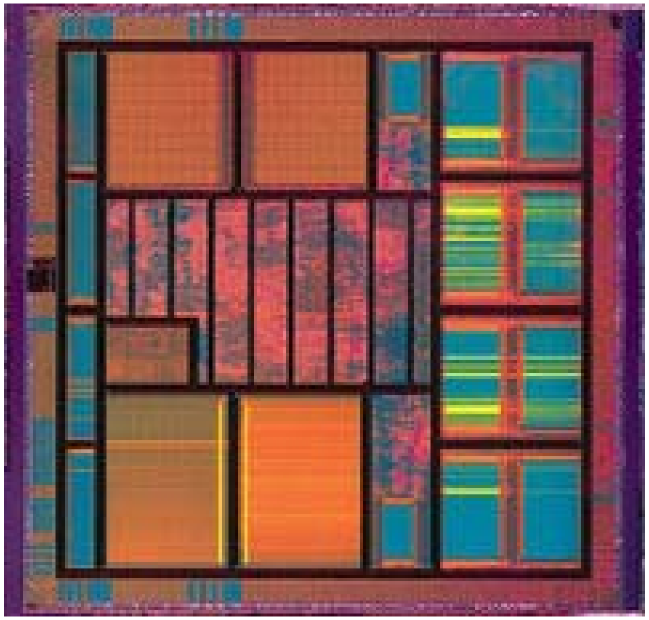
c) Results at T = 1.2 (α = 2.0567)

d) Results at T = 0.8 (α = 1.515)



Physical Design of Computers

- Optimization problems in Comp. design
  - Partitioning circuits into groups to fit on chip
  - Placement of circuits on chip
  - Wiring of circuits on chip
- Goal to optimize performance of system without compromising any design stage



Physical Design of Computers continued…

- Partitioning
  - Number of circuits in each partition must fit into package
  - Number of signals crossing boundaries minimized
- Placement
  - Minimize length of connections (reduce signal propagation time)
  - Minimize congestion (overcrowding)
- Wiring
  - Minimize wire lengths used
  - Minimize source of noise

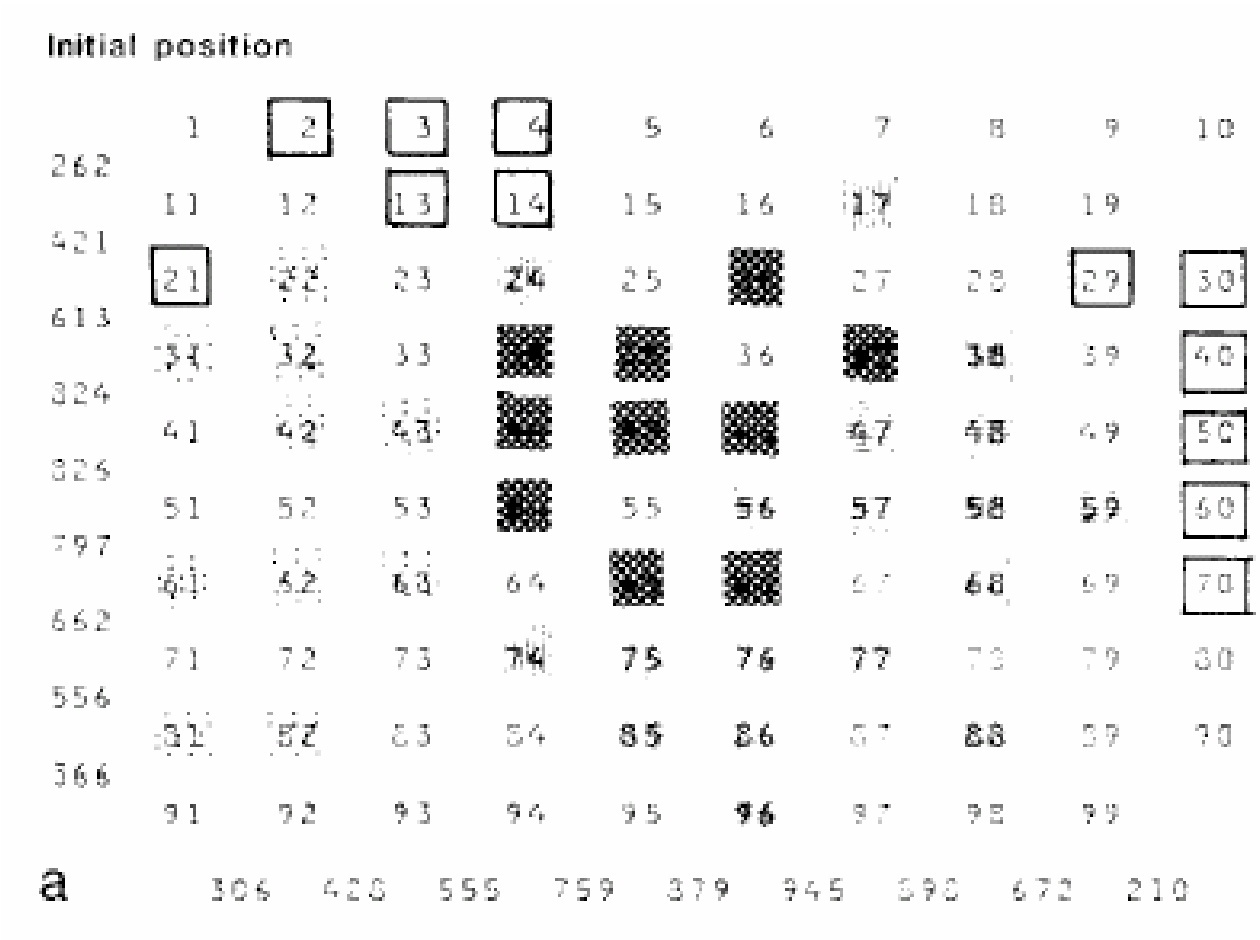
SA: Placement Problem

- **Problem**: Placement of 98 chips on IBM 3081 processor with 100 sites (10 x 10 grid)
- Re-arrangement moves involve switch between two chips or switch between chip and vacancy
- Histograms used to keep track of congestion and wire length by scoring boundary crossing on grid
  - Minimum one wire per boundary crossed
  - Sum of horizontal bins gives lower bound of horizontal length
  - Sum of vertical bins gives lower bound of vertical length
  - Construction of objective function

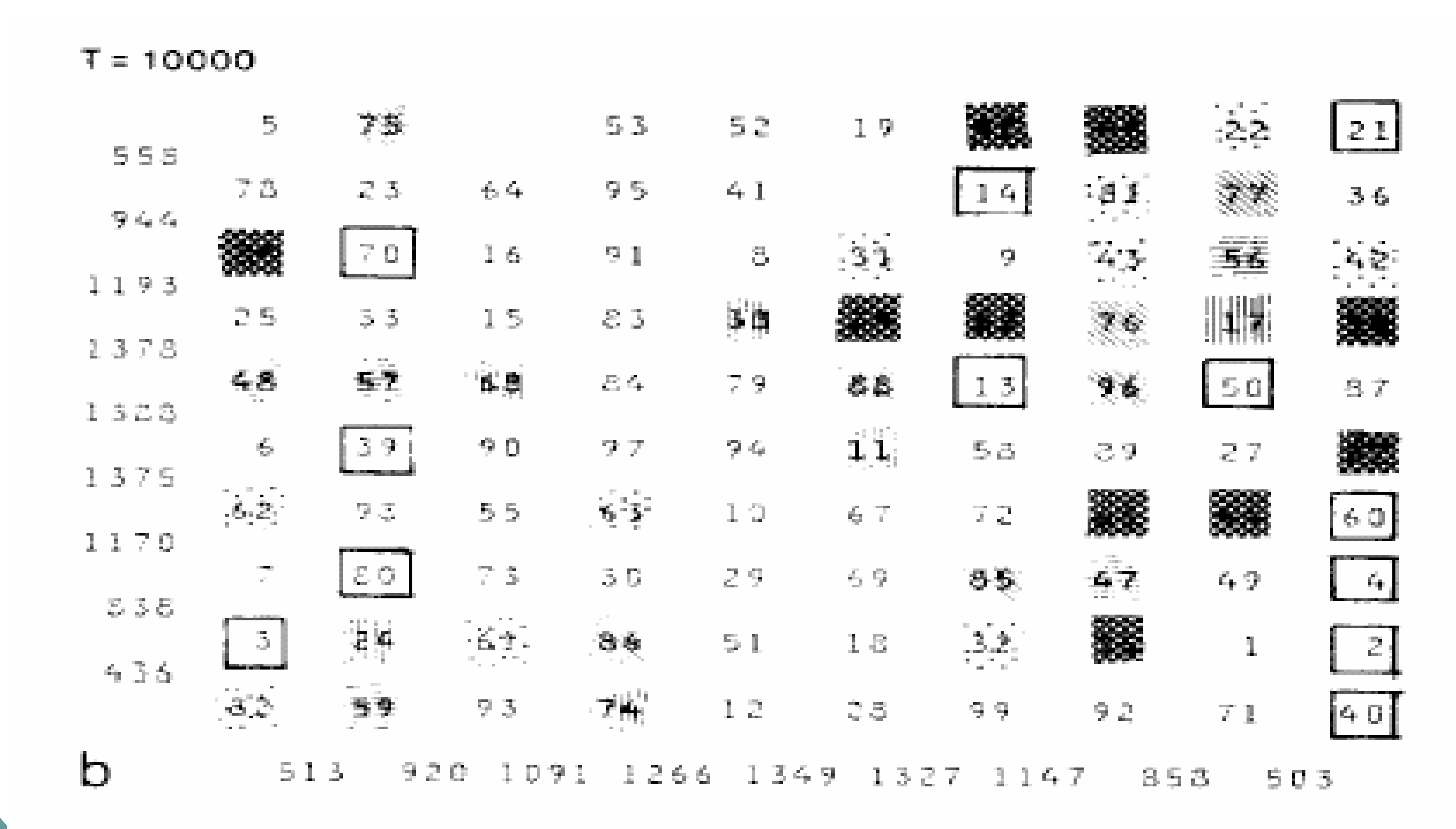
SA: Placement Problem continued…

- Chips are numbered from 1 to 99 (without chip 20)
- Dark squares represent adder chips
- Squares with ruled lines represent chips that supply data to adder chips
- Lightly dotted squares represent chips that perform logical arithmetic (and, or, etc.)
- Open squares represent general-purpose register chips

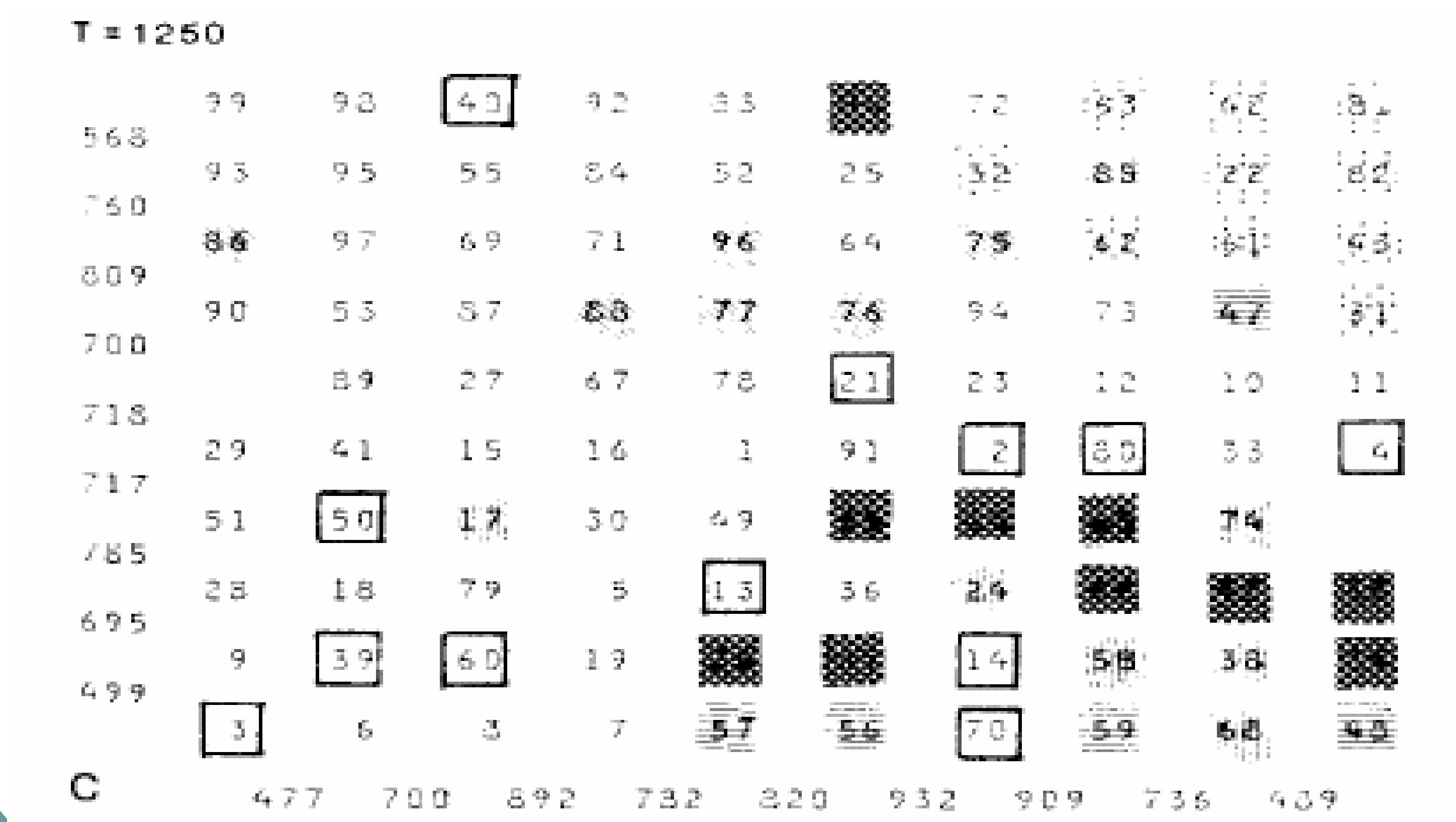
SA: Placement Problem continued…



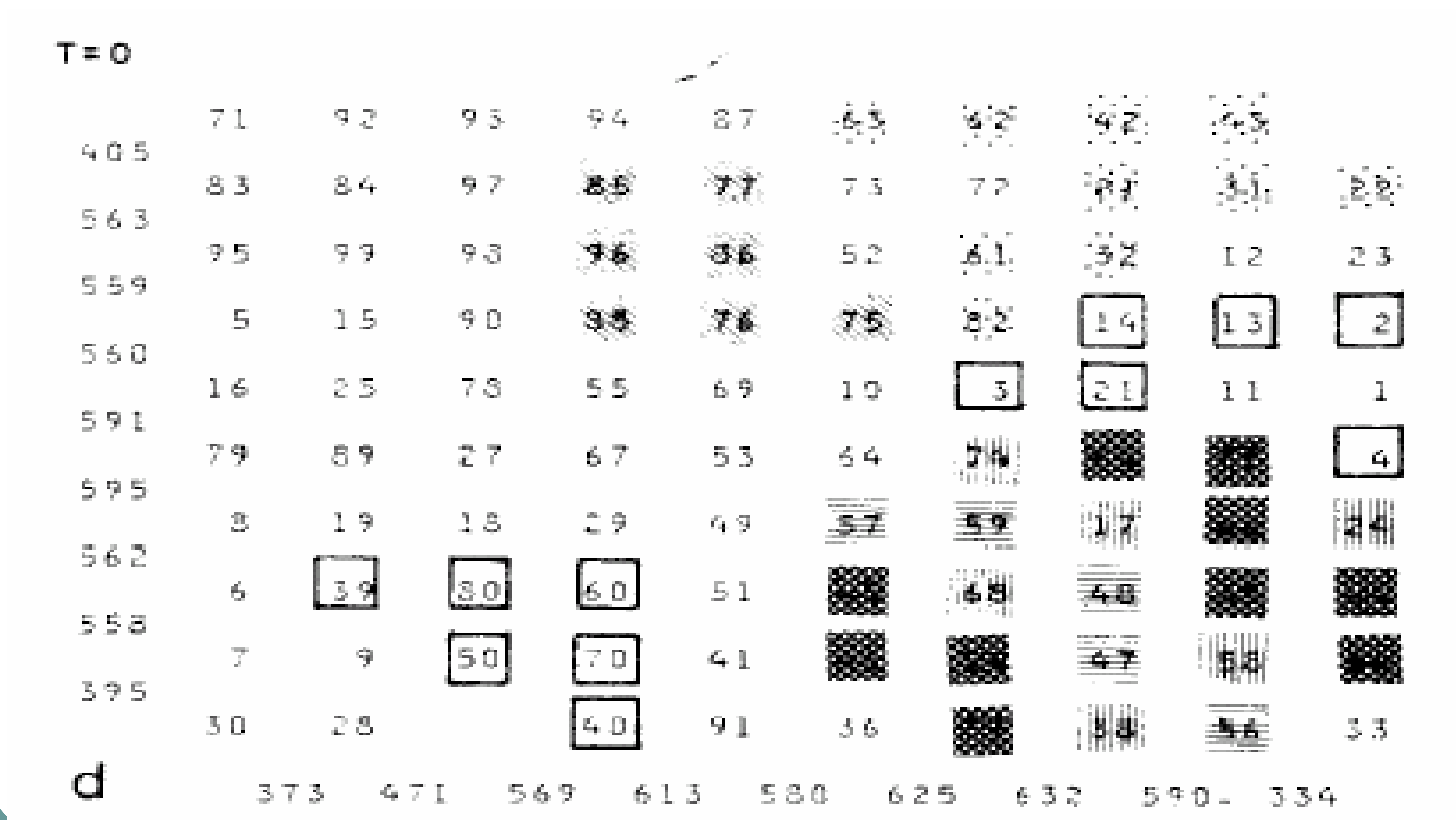
SA Results: Placement Problem



SA Results: Placement Problem
continued…



SA Results: Placement Problem continued…



SA Results: Placement Problem continued…

- Observed decrease in congestion as T was decreased in SA
- Observed decrease in wire length as T was decreased in SA (minimization of wire length)

Conclusion

- Simulated annealing with Metropolis algorithm is effective heuristic technique
- Require known initial configuration, objective function, random number generator, and temperature schedule (annealing)
- Success with finding near-optimal solutions for NP-C problems (Traveling salesman)
- Success with optimizing computer design

References

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